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AIRPLANE AIR CONDITIONING
ENGINEERING DATA

THERMODYNAMICS



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AIRPLANE AIR CONDITIONING ENGINEERING DATA
THERMODYNAMICS

INTRODUCTION

This report is intended to present a brief and factual review of thermodynamics as applied to subsonic air flow systems for aircraft. It is generally limited to air in the gaseous state, however, certain effects of entrained water vapor are included. Incidental heat transfer effects due to potential differences of duct walls within a given flow system are considered to be negligible for aircraft problems. Useful thermodynamic and psychrometric charts are also provided.

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1. THERMODYNAMICS:

1.1 Thermodynamic Terms: The following is a list of symbols which are used to represent the indicated thermodynamic terms. For purposes of calculation, all quantities are expressed in consistent foot, pound, and second basic units:

p = Static pressure (lb/sq ft)

q = Dynamic pressure (lb/sq ft) $q = \frac{\rho v^2}{2}$

V = Volume (cu ft)

v = Specific volume (cu ft/lb)

γ = Specific weight (lb/cu ft) $\gamma = \frac{1}{v}$

ρ = Mass density (slugs/cu ft) $\rho = \frac{\gamma}{g}$

σ = $\frac{\rho}{\rho_o}$ density ratio referred to sea level air

w = Mechanical energy of work (transient form) (ft-lb/lb)

u = Mechanical energy (stored form) (ft-lb/lb)

u_p = Mechanical potential energy (ft-lb/lb)

u_k = Mechanical kinetic energy (ft-lb/lb)

t = Temperature (degrees Fahrenheit, °F.)

T = Temperature (degrees Rankine, °R.) $T = 460 + t$

hp = Horsepower

ω = Angular velocity

E = Energy in general

J = Mechanical equivalent of heat (778 ft-lb/Btu)

A = Heat equivalent of mechanical energy ($\frac{1}{778}$ Btu/ft-lb)

g = Gravity constant (32.174 ft/sec/sec)

v = Velocity (ft/sec) $v = 1.467 v_m$

v_m = Velocity (miles/hr) $v_m = 0.6818 v$

c_p = Specific heat at constant pressure (Btu/deg/lb)

c_v = Specific heat at constant volume (Btu/deg/lb)

$H = U + Apv$ = Enthalpy (Btu/lb)

- Q = Heat from or to an outside reservoir (Btu/lb)
 U = Internal (molecular) energy (Btu/lb)
 S = Entropy = The measure of the degradation of energy available for work (Btu/deg/lb)
 r = Radius
 η = Efficiency
 M = Mach number

The following are physical constants for standard atmospheric air. These constants are approximations but are of value in later calculations:

- $c_p = 0.241$ Btu/deg/lb of air (specific heat at constant pressure)
 $c_v = 0.173$ Btu/deg/lb of air (specific heat at constant volume)
 $k = \frac{c_p}{c_v} = 1.4$ ratio of specific heats for air
 $R = 53.33$ ft-lb/deg/lb of air

For standard N.A.C.A. atmosphere at sea level the following values hold true:

- $p_o = 2116.23$ lb/sq ft (14.697 lb/sq in. = 29.921 "Hg)
 $t_o = 58.4^\circ\text{F}$ (Approximate value = 59°F)
 $T_o = 518.4^\circ\text{R}$ (Approximate value = 519°R)
 $\rho_o = 0.002378$ slugs/cu ft
 $\sigma_o = \frac{p_o}{\rho_o} = 1$
 $\gamma_o = \frac{p_o}{\rho_o} = 0.07651$ lb/cu ft = g_o
 $v_o = 13.07$ cu ft/lb = $\frac{1}{\gamma_o}$

2. DEFINITIONS:

- 2.1 Adiabatic Flow Systems are those in which external work is performed on or by the fluid without heat transfer.
 2.2 Heat Transfer Systems (not to be confused with mixing processes) are those flow systems in which heat is added-to, or subtracted-from, the passage walls by the fluid and no work is involved. Thus, the total energy of the fluid with respect to the passage walls must change and a temperature gradient must exist. Heat exchangers and radiators are examples.
 2.3 Mechanical Energy Transfer Systems are those in which work is done on or by the passage wall by means of the working fluid and no heat is transferred.

2.4 Static (Stream) Temperature is that temperature sensed by a measuring means at rest with respect to the fluid measured.

Symbol: T_s = Absolute temperature, $^{\circ}R$

t_s = Fahrenheit temperature, $^{\circ}F$

2.5 Total (Impact, Stagnation) Temperature is equal to the stream temperature of a moving fluid plus an increment corresponding to the kinetic energy of the fluid, with respect to the sensing means.

Symbol: T_t, t_t

$$\text{Equation: } T_t = T_s + \frac{v_r^2}{2g \cdot 778 \cdot c_p}$$

$$t_t = t_s + \frac{v_r^2}{2g \cdot 778 \cdot c_p}$$

Where: v_r = Average relative fluid velocity at sensing point
(in ft/sec).

2.6 Static (Stream) Pressure is that absolute pressure sensed by a measuring means at rest with respect to the fluid measured.

Symbol: p_s

2.7 Total (Impact) Pressure is equal to the static (stream) absolute pressure plus an increment corresponding to the kinetic energy of the fluid, with respect to the sensing means.

Symbol: p_t

$$\text{Equation: } \left(\frac{p_t}{p_s} \right)^K - 1 = \frac{v_r^2}{2g \cdot 778 \cdot c_p \cdot T_s} \quad * \text{ (for air)}$$

* The general form $\left(\frac{p_2}{p_1} \right)^{.283 - 1}$ commonly appears in thermodynamic equations for

air. It is widely known as a "Y" factor, and tables are available for "Y" as a function of pressure ratio.

$$K = \frac{\frac{c_p}{c_v} - 1}{\frac{c_p}{c_v}} = \frac{k-1}{k} = 0.283$$

Where: $\frac{c_p}{c_v} = k = 1.395$ for air at -50° to $+900^{\circ}F$

- 2.8 "Adiabatic" Efficiency (general) is expressed in terms of the change in total pressure and the change in specific energy of the working fluid. It is so expressed that an isentropic process would have 100% efficiency. Definition equations appear later in this section.
- 2.9 Isentropic Processes are those in which a change of state of the fluid takes place without fluid friction, heat transfer, or turbulence. It thus represents an "ideal" adiabatic process. In an isentropic process, the entropy of the fluid remains constant.
- 2.10 A Boundary Layer is that portion of a flowing fluid adjacent to a solid body which has virtually no relative motion with respect to that body. Such layers of stagnant fluid occur in all flow systems, but the relative thickness varies widely.
- 2.11 Mach Number (M) is the ratio of a fluid relative velocity to the velocity of sound at an established point. For air, the velocity of sound equals $49\sqrt{T_g}$ feet per second.

3. BASIC PRINCIPLES:

- 3.1 In an adiabatic flow system, the total temperature of a fluid with respect to the passage walls (or, therefore, with respect to any observer at rest with respect to those walls) remains constant throughout the system.
- 3.2 In an adiabatic flow system, the stream temperature of the fluid is varied by fluid friction and turbulent losses, but the total temperature, as noted above, does not change.
- 3.3 In a heat transfer system, the stream temperature usually varies, but the total temperature must change by an amount corresponding to the specific energy transfer to, or from, the fluid.
- 3.4 In a mechanical energy transfer system, the stream temperature usually varies, but the total temperature must change by an amount corresponding to the specific energy added-to, or subtracted-from, the fluid.
- 3.5 In a heat transfer system, the total pressure of the fluid cannot be increased, and is usually decreased, regardless of whether heat is added or subtracted.
- 3.6 In a mechanical energy transfer system, the total pressure of the fluid may be increased by the addition of energy, but this is not necessarily so.
- 3.7 In calculating density and specific volume, static temperatures and static pressures must be used.
- 3.8 If passage wall temperatures differ appreciably from fluid total temperature, the means for measuring fluid temperature must be shielded from wall radiation.
- 3.9 In an adiabatic flow system, conversions between kinetic and potential energy may occur. Thus, an increase in velocity will decrease the static pressure and static temperatures. However, the total temperature is unchanged, and the total pressure decreases only by an amount related to the fluid friction and turbulence losses.
- 3.10 Boundary layer temperatures very nearly equal the corresponding total temperature of the fluid flowing at low values of kinetic energy of the fluid unless passage wall heat transfer imposes a temperature gradient.

3.11 Boundary layer pressure is equal to static pressure.

3.12 In any flow system, the algebraic sum of heat and mechanical energy transfers, in any portion of the system, is proportional to the mass flow and the specific energy change, as measured by the change in total temperature. Nothing need be known about the physical process to make this determination. (A restatement of the First Law of Thermodynamics).

3.13 In considering any mechanical energy transfer process, the frame of reference must be established. For instance, consider a ram duct in an airplane in flight. With respect to the airplane, the duct operates as an adiabatic flow system, whereas, with respect to a ground observer, the duct operates as a mechanical energy transfer system. The significant difference lies in the motion of the duct wall with respect to the observer in the latter case.

3.14 A special case of mechanical energy transfer occurs when flow is through a centrifugal field induced by angular motion of the duct. In such case, an energy-potential difference exists between portions of the duct wall not equidistant from the center of rotation. In such a system, choosing a point "a" on the rotating duct as a frame of reference, the specific energy of the fluid with respect to some other point "b" must equal the specific energy of the fluid with respect to "a" minus the potential difference of "b" with respect to "a". This is of some importance in calculating surface temperatures.

Thus:

$$E_a - E_b = \frac{(r_b \omega)^2}{2g} - \frac{(r_a \omega)^2}{2g}$$

Where: E_a = specific energy of fluid with respect to "a" in ft-lbs/lb.

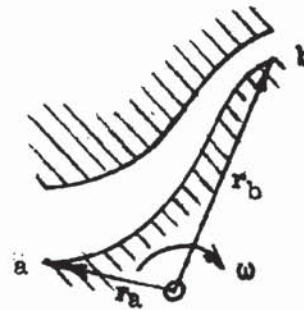
E_b = Specific energy of fluid with respect to "b" in ft-lbs/lb.

r_a = radius from center of rotation to "a" in feet

r_b = radius from center of rotation to "b" in feet

ω = angular velocity in rad/sec

$g = 32.17 \text{ ft/sec}^2$



In the above, E_a and E_b refer to the same portion of fluid at the same time.

4. THERMODYNAMIC CHARTS:

4.1 Although air behaves approximately as a perfect gas, for accurate work covering a wide range of temperatures, a thermodynamic chart is desirable. Such charts are based on actual measurements of air properties.

4.2 The change in specific energy content of a fluid may be determined with the aid of the applicable chart. All that need be known is the fluid temperature. If total energy (kinetic + internal) is desired for any state, use total temperature. If internal energy is desired, use static temperature.

- 4.3 The chart may also be used to convert static and total pressures and temperatures.

Thus:

Let: ΔE = Change in specific energy
content in ft-lbs/lb

v_r = Air velocity in ft/sec

$$\Delta E = \frac{v_r^2}{2g}$$

If either T_s or T_t is known, its corresponding specific energy may be found on the chart. ΔE_t is then added (if T_s is known), or subtracted (if T_t is known), to find the specific energy corresponding to the unknown value. For values of v_r approaching or exceeding sonic speeds, the chart should be used instead of the equation.

- 4.4 In the thermodynamic chart included in this information report, (Fig. 4) the ordinate has scales of temperature, enthalpy, and specific energy. Once any one of these quantities is known, the other two are determined. The abscissa is entropy. Cross-plotted are lines of absolute pressure and specific volume. Properties of air may be grouped thus:

- (Temperature
- a. - (Enthalpy
- (Specific energy
- b. Density
- c. Pressure
- d. Entropy

If quantities in any two of the above groups are known, all other properties of the air at the established state may be determined from the chart.

5. ENTROPY-ENTHALPY CHART FOR AIR:

- 5.1 Introduction: In computing or plotting the performance of a compressor, supercharger, regulating valve, air conduit, etc., a thermodynamic chart wherein the change of state of air can be visualized is of great help. The most practical is the Entropy-Enthalpy Chart.

The two coordinates are Enthalpy and Entropy. Enthalpy is the sum of the internal (molecular) and displacement work energies in one pound of air at rest, measured from an arbitrary zero level. Entropy is a measure of degradation of the "available-for-work" energy, measured from an arbitrary zero point.

By plotting the calculated or experimental values of pressure and temperature of the air at various points in its flow, a chart is obtained from which all the other values which determine the state of the air can be read directly. The specific volume, the specific weight, the density ratio, and the energy can be directly determined from such a chart.

5.2 Chart Units (See Fig. 2): As this chart will be used for investigating aircraft devices, it is advisable to express and plot constant pressure lines in in. Hg, densities as ratios, temperatures in °F, and energy and work in British thermal units or their equivalent ft-lb mechanical work units.

5.3 Use of Chart (See Fig. 3): The work of compressor shaft is transferred by means of the impeller to the air as compression energy and heat energy. The sum of these two energies is called enthalpy and is measured as heat in Btu.

To compress the air at 100% efficiency in a heat insulated air compressor from its initial state p_A pressure and t_A temperature to p_B pressure and t_B temperature, the compression work is A to B enthalpy units. Perfect compression means that none of the available work energy is degraded; entropy stays unchanged. A change taking place with constant entropy is represented by a vertical line. A and B are in isentropic relationship to each other on such a line. The minimum limiting compressed air temperature in a heat insulated compressor is therefore t_B °F.

Testing an actual compressor, we find that at the same p_B pressure, the observed air temperature is t_C °F., which is higher than the temperature for perfect compression efficiency t_B °F. This excess heating of the air requires B to C enthalpy units or its equivalent extra foot-pounds of shaft work input. Thus, the input energy level has been raised from B, the theoretically perfect efficiency level to the value C.

The shaft efficiency of the compressor therefore is:

$$\eta = \frac{AB}{AC} = \frac{u_{AB}}{u} = \frac{H_{AB}}{H} = \frac{t_B - t_A}{t_C - t_A}$$

The excess or wasted energy BC may also be expressed as:

$$BC = H_C - H_B = T_A (S_C - S_B)$$

By plotting all test data on the chart, a complete picture of work energy transfer may be visually presented and any weak spots in design can be detected.

5.4 Correlation of Chart Units: To construct the entropy-enthalpy chart it is necessary to know the thermodynamic laws which apply to air. In this investigation air will be assumed to be a dry, perfect gas.

The Gas Law

$$pv = RT \quad \text{--- (1)}$$

$$d(pv) = RdT \quad \text{--- (2)}$$

$$pdv + vdp = RdT \quad \text{--- (3)}$$

Enthalpy (H)

$$H = U + Apv$$

$$dH = dU + Ad(pv) \text{ - - - - - (4)}$$

$$= dU + ARdT \quad (\text{from equation (2)})$$

$$\text{but } dU = c_v dT$$

$$\text{Then: } dH = c_v dT + ARdT$$

$$\text{Or: } dH = (c_v + AR)dT \text{ - - - - - (5)}$$

$$\text{Also: } dH = c_p dT = c_p dt \text{ - - - - - (6)}$$

5.4.1 Specific Heat at Constant Pressure (c_p): The value of specific heat varies with temperature change and also with pressure change. At constant atmospheric pressure, the Parkington and Shilling Test (Eshbach Handbook of Engineering Fundamentals, page 7-17) gives:

$$c_p = 0.1699 + 0.0000033T + 0.0000000032T^2$$

At constant 140°F. temperature, according to Ford (Compressor Theory and Practice):

$$c_p = 0.2414 - 0.00002 p$$

where p = pressure in lb/sq in.

Figure I, page 7-18 of Eshbach, shows curves of values of c_p with variation of temperature.

In aircraft applications, the temperature changes of air are not excessive, but the humidity of the air is quite variable. In engineering investigations, these secondary influences may be neglected and a mean c_p value used.

With $c_p = 0.241$ as the selected constant value, equation (6) becomes:

$$\Delta H = H_B - H_A = c_p (t_B - t_A) \text{ - - - - - (7)}$$

That is, change in enthalpy is proportional to change in temperature. The enthalpy may be expressed in foot-pounds mechanical energy or work units, since $1 \text{ Btu} = 778 \text{ ft-lb}$.

Equation (7) then becomes:

$$H_B - H_A = .241 (t_B - t_A) = \frac{1}{778} (u_B - u_A) \text{ - - - - (8a)}$$

then,

$$(u_B - u_A) = 187.5 (t_B - t_A) \text{ - - - - - (8b)}$$

5.4.2 Entropy: The first law of thermodynamics states that work-and heat-energies are equivalent. This law does not define the change of state of air.

The entropy concept is introduced to enable the determination of the maximum possible heat-work energy transfer in certain ideal processes.

Some of the many definitions of entropy are:

Entropy is a measure of thermodynamic degeneration of a heat-work energy transfer.

Entropy is determined by the quantity of heat in the body and its temperature.

Entropy multiplied by the minimum available absolute temperature of a given heat-work process equals the portion of the total energy transfer which is rejected or made unavailable for useful work.

5.4.3 The Mathematical Definition of Entropy-Change:

$$dS = \frac{dQ}{T} \text{ ----- (9)}$$

$$dQ = dH - Avdp$$

$$= dU + Apdv$$

$$T \cdot ds = dH - Avdp$$

For constant pressure process $dp = 0$

$$(ds)_p = \frac{dH}{T} \text{ ----- (10)}$$

or

$$dH = T \cdot ds \text{ ----- (10a)}$$

$$(H_C - H_B) = \int_B^C T \cdot ds \quad (\text{See Figure 3, page 30})$$

That is, the enthalpy change is equal to the area below the B - C line and measured from the absolute zero temperature base line.

5.4.4 Energy Flow: The work energy put into the driven compressor shaft is transferred to the air by means of the impellers.

$$w_{in} = \text{Shaft input work (ft-lb)}$$

$$v = \text{Absolute velocity of air through the compressor (ft/sec)}$$

$$v_r = \text{Relative velocity of air in impeller (ft/sec)}$$

$$v_p = \text{Peripheral velocity of impeller (ft/sec)}$$

Note: Subscript ₁ denotes inlet conditions
 Subscript ₂ denotes outlet conditions

In a heat insulated compressor

$$Q = 0$$

Energy balance of inlet and outlet air:

$$w_{in} + \frac{v_1^2}{2g} + p_1 v_1 + JU_1 = \frac{v_2^2}{2g} + p_2 v_2 + JU_2 \text{ --- (11)}$$

Solving for input energy and collecting terms:

$$w_{in} = \frac{v_2^2 - v_1^2}{2g} + (p_2 v_2 - p_1 v_1) + J(U_2 - U_1)$$

That is, work input = change in kinetic energy plus displacement work plus internal energy increase of the air.

But,

$$pv + JU = JH,$$

then,

$$w_{in} = \frac{v_2^2 - v_1^2}{2g} + J(H_2 - H_1)$$

or,

$$H_2 - H_1 = \frac{w_{in}}{J} - \frac{v_2^2 - v_1^2}{J2g}$$

That is, the enthalpy change of air through the compressor is proportional to the work input less the change in kinetic energy between outflowing and inflowing air.

In most cases the kinetic energy change is negligible. Therefore, the enthalpy change is directly proportional to the work input and may be determined from the simplified expression:

$$H_2 - H_1 = Aw_{in}$$

5.4.5 Determination of Scales and Constant Pressure and Constant Density Lines

5.4.5.1 Enthalpy, Energy, Temperature Scales: These three scales are convertible through the use of equations (8a) and (8b), as each expresses energy in different units.

$$\begin{aligned} H_B - H_A &= .241 (t_B - t_A) && \text{in (Btu)} \\ &= \frac{(u_B - u_A)}{778} && \text{in (Btu)} \end{aligned}$$

$$u_B - u_A = 778 (H_B - H_A) \quad \text{in (ft-lb)}$$

$$= 187.5 (t_B - t_A) \quad \text{in (ft-lb)}$$

$$t_B - t_A = \frac{(H_B - H_A)}{.241} \quad \text{in } (^{\circ}\text{F or } ^{\circ}\text{R})$$

$$= \frac{(u_B - u_A)}{187.5} \quad \text{in } (^{\circ}\text{F or } ^{\circ}\text{R})$$

Each of these scales has uniform divisions which are convertible each to the other two. The vertical lengths represent energy changes.

5.4.5.2 Entropy Scale: The selection of a uniformly divided scale for the enthalpy changes makes it necessary that the entropy scale be correlated through the following relationships:

by definition,

$$dS = \frac{dQ}{T}$$

and

$$dQ = c_v dT + A p dv$$

$$dQ = c_p dT - A v dp$$

$$pv = RT$$

by substitution,

$$dS = c_v \frac{dT}{T} + A p \frac{dv}{T} = c_v \frac{dT}{T} + AR \frac{dv}{v}$$

$$\left(\frac{p}{T} = \frac{R}{v}\right)$$

and,

$$dS = c_p \frac{dT}{T} - A v \frac{dp}{T} = c_p \frac{dT}{T} - AR \frac{dp}{p}$$

Integrating the above equations between the limits 1 and 2:

$$\int_1^2 dS = c_v \int_1^2 \frac{dT}{T} + AR \int_1^2 \frac{dv}{v}$$

$$\int_1^2 dS = c_p \int_1^2 \frac{dT}{T} - AR \int_1^2 \frac{dp}{p}$$

or

$$S_2 - S_1 = c_v \log_e \frac{T_2}{T_1} + AR \log_e \frac{v_2}{v_1} \text{ --- (13)}$$

$$S_2 - S_1 = c_p \log_e \frac{T_2}{T_1} - AR \log_e \frac{p_2}{p_1} \text{ --- (14)}$$

Since density is inversely proportional to volume,

$$\frac{v_2}{v_1} = \frac{\rho_1}{\rho_2} = \frac{\sigma_1}{\sigma_2}$$

and noting that,

$$\log_e \frac{v_2}{v_1} = \log_e \frac{\sigma_1}{\sigma_2} = -\log_e \frac{\sigma_2}{\sigma_1}$$

then equation (13) becomes

$$S_2 - S_1 = c_v \log_e \frac{T_2}{T_1} - AR \log_e \frac{\sigma_2}{\sigma_1} \text{ --- (13a)}$$

Equations (13), (13a) and (14) are useful in plotting constant volume, constant density and constant pressure curves.

In equation (13a) and (14), for any given pair of values of $\frac{\sigma_2}{\sigma_1}$ and $\frac{p_2}{p_1}$ on any $T = \text{a constant line}$,

$$\log_e \frac{T_2}{T_1} \text{ will equal zero; } AR \log_e \left(\frac{\sigma_2}{\sigma_1} \right) \text{ and } AR \log_e \left(\frac{p_2}{p_1} \right)$$

are constant, therefore all constant density ratio curves are displaced parallel in the horizontal direction. The same is true for the constant pressure lines.

For $\frac{\sigma_2}{\sigma_1} = 1$, the value of $\log_e \frac{\sigma_2}{\sigma_1}$ is zero. The equation (13a) becomes,

$$\begin{aligned} (S_2 - S_1) &= c_v \log_e \frac{T_2}{T_1} \\ &= c_v (2.30258 \log_{10} \frac{T_2}{T_1}) \quad (c_v = .173) \end{aligned}$$

then,

$$(S_2 - S_1)_\sigma = 0.39835 (\log_{10} T_2 - \log_{10} T_1) \text{ --- (15)}$$

5.4.5.3 Constant Pressure Lines: The same reasoning, as applied to the constant density ratio lines, reduces equation (14) to:

$$(S_2 - S_1)_p = c_p \log_e \left(\frac{T_2}{T_1} \right)$$

$$= c_p (2.30258 \log_{10} \frac{T_2}{T_1}) \quad (c_p = .241)$$

$$= 0.55492 (\log_{10} T_2 - \log_{10} T_1) \quad \text{--- (16)}$$

Selecting, arbitrarily, sea level air (NACA Standard) as the origin for the scale of $(S_2 - S_1)$ values, $T_0 = 460 + 59 = 519^\circ\text{R.}$

Making use of equations (15) and (16) for determining values of $(S_2 - S_0)_\sigma$ and $(S_2 - S_0)_p$, the following tabulation determines the forms of the constant density ratio lines and the constant pressure lines, where

$$T_1 = 59^\circ + 460 = 519^\circ\text{R.} = T_0$$

$$\log_{10} T_1 = 2.71517$$

$t_2^\circ\text{F.}$	$T_2^\circ\text{R.}$	$\log_{10} T_2$	$\log_{10} T_2 - \log_{10} T_1$	$(S_2 - S_0)_\sigma$	$(S_2 - S_0)_p$
500	960	2.98227	.26710	.10640	.14824
450	910	2.95904	.24387	.09715	.13535
400	860	2.93450	.21933	.08736	.12173
350	810	2.90849	.19332	.07700	.10729
300	760	2.88081	.16564	.06598	.09193
250	710	2.85126	.13609	.05421	.07553
200	660	2.81954	.10437	.04157	.05793
150	610	2.78533	.07016	.02794	.03894
100	560	2.74819	.03302	.01315	.01833
50	510	2.70757	-.00760	-.00303	-.00422
0	460	2.66276	-.05241	-.02090	-.02908
-50	360	2.55630	-.15887	-.06330	-.08817
-150	310	2.49136	-.22381	-.08916	-.12421

$$\text{Equation (15)} - (S_2 - S_0)\sigma = 0.39835 (\log_{10} T_2 - \log_{10} T_1)$$

$$\text{Equation (16)} - (S_2 - S_0)_p = 0.55492 (\log_{10} T_2 - \log_{10} T_1)$$

These two equations give the value of entropy measured positively to the right and negatively to the left from the arbitrary origin for varying temperature changes above or below the arbitrarily chosen temperature 0°F. or 460°R.

5.4.6 Scale of Constant Density Ratio and Constant Pressure Curves: These curves are displaced parallel in the "S" direction, that is, in the horizontal direction along a line, $T = \text{a constant}$. On such a line, $T_2 = T_1$, $\frac{T_2}{T_1} = 1$

$$\text{and } \log_e \frac{T_2}{T_1} = 0.$$

Then equations (13a) and (14) become,

$$(S_2 - S_1)_T = -AR \log_e \frac{\sigma_2}{\sigma_1} \text{ - - - - - (13b)}$$

$$(S_2 - S_1)_T = -AR \log_e \frac{P_2}{P_1} \text{ - - - - - (14a)}$$

The length of one $\log_{10}$ scale extends from 1 to 10 or 10 to 100, etc., values in each case are:

$$\frac{\sigma_2}{\sigma_1} = 10$$

$$\frac{P_2}{P_1} = 10$$

$$\log_e 10 = 2.30258$$

$$(S_2 - S_1)_T = -AR(2.30258 \log_{10} \frac{\sigma_2}{\sigma_1}) = -AR(2.30258 \log_{10} \frac{P_2}{P_1})$$

$$(S_2 - S_1)_T = -0.1578 \log_{10} \frac{\sigma_2}{\sigma_1} = -0.1578 \log_{10} \frac{P_2}{P_1} \text{ - - - - - (14b)}$$

i.e., the two ratio scales 0.1 to 1.0 increase toward the left and are .1578 "S" units in length. Therefore, the density ratio and the pressure ratio scales are the same and one logarithmic scale can serve for both ratios. Density ratio can be read as horizontal distance from the density ratio = 1 line and read off as a decimal on the logarithmic scale from the value 1. Likewise, pressure ratio can be read off as a distance measured horizontally from the standard pressure line 29.92 "Hg, and read on the logarithmic scale from the value 1.

Constant pressure lines are drawn on the chart, but to avoid confusion of lines only the one density ratio line is drawn through point of standard NACA sea-level air,

$$\sigma = \frac{P_0}{P_0} = 1$$

Each logarithmic scale ($\log_{10}$) is .1578 "S" units in length.