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Statistical Stability Assessment

RATIONALE

Engine stability analysis and assessment, and the allocation of stability margins, form integral parts of the design and development of every gas turbine engine program. The existing guidelines for stability assessments are documented in ARP1420 and AIR1419, which have been widely utilized throughout industry. However, the existing guidelines do not recognize the statistics of operational demands and can therefore potentially over emphasize low probability events. Applying statistical techniques allow an assessment of the probability of encountering a destabilizing event and thus enable a risk assessment to be made for inlet/engine compatibility. The application of such techniques form the basis of the methodology discussed in this Aerospace Information Report.

This document contains information that is fundamental to the discipline of inlet-engine compatibility and is expected to remain stable for the foreseeable future.

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FOREWORD

Engine stability analysis and assessment, and the allocation of stability margins form integral parts of the design and development of every gas turbine engine program. The existing guidelines for stability assessments are documented in ARP1420 and AIR1419, which have been widely utilized throughout industry. A key aspect of the stability assessment process involves assessing the statistical aspects of stability and the probabilistic nature of stability limits. As a result, there has been significant progress in developing and applying Monte Carlo techniques to the stability assessment process. A discussion of such techniques forms the basis of the methodology discussed in this Aerospace Information Report.

The overall approach outlined in the document is referred to as the Statistical Stability Assessment or SSA. Information is provided on the various models and methods that make up the SSA process. The validity and application of the approach is examined.

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1. SCOPE

This SAE Aerospace Information Report (AIR) provides a methodology for performing a statistical assessment of gas-turbine-engine stability-margin usage. Consideration is given to vehicle usage, fleet size, and environment to provide insight into the probability of encountering an in-service engine stall event. Current industry practices, such as ARP1420, supplemented by AIR1419, and engine thermodynamic models, are used to determine and quantify the contribution of individual stability threats. The statistical technique adopted by the S-16 committee for performing a statistical stability assessment is the Monte Carlo method (see Applicable References 1 and 2). While other techniques may be suitable, their application is beyond the scope of this document. The intent of the document is to present a methodology and process to construct a statistical-stability-assessment model for use on a specific system and its mission or application.

1.1 Purpose

The purpose of the Statistical Stability Assessment (SSA) is to provide a quantitative tool for optimizing engine stability margin requirements and provide greater visibility of off-design conditions. It can be applied at the inception of a new project to perform trade studies and support risk assessments; or to a mature project to assess the impact/risk of modifications or changes in operational usage.

This document provides guidelines on the procedures and data that are required to perform the statistical stability assessment. A number of illustrative examples are presented.

1.2 Field of Application

The compatibility of a gas turbine engine with its operating environment is a major concern in all gas turbine applications. The installation of an engine, be it in an aircraft, ship, or ground facility, determines the quality of airflow provided to the compression system of the engine and has a significant influence in the setting of the engine stability margin. This has a direct impact on the performance, operability, and mechanical integrity of the installed engine. It is the operability aspects that the SAE S-16 committee has devoted its main effort: understanding and accounting for the effects of intake flow-field distortion on engine stability.

The current industry guideline for assessing the effect of intake flow-field distortions has been the subject of previous SAE publications, notably ARP1420 and AIR1419. The loss of compression-system stability margin due to inlet flow distortion is combined with the losses from other factors to produce a stability margin stack-up. Traditionally, stack-ups are performed at a number of flight conditions that are judged to be the most critical by the engine company, based on information available from the airframe company and the operator. Destabilizing factors are judged to be either random or non-random and summed accordingly to provide an estimate of the likelihood of encountering engine instability at the chosen flight condition. In practice, stability margins are usually set such that engine instability is predicted not to occur.

While this methodology continues to serve the industry well, it is based on the assumption that the random factors are normally distributed and also offers limited insight into the likely stall rate for a fleet of aircraft. Further, the growing number of factors that are accounted for in the stability stack-up, the increased use of variable geometry, and advanced control modes have made it potentially more difficult to identify a small number of critical conditions that would indicate the engine to be truly stall free.

This document presents a statistical approach to model the engine operating environment with a view to identifying critical operating points and predicting a probability of compression system instability. Consideration is given to atmospheric variations, vehicle usage, and installation effects. The resulting stability assessment can provide improved insight into engine stability and be used to support trade studies, risk assessments, and design optimization.

To adequately capture the engine operating environment a detailed knowledge of the aircraft utilization is required. This extension of the stability assessment, therefore, demands much greater participation from both the operator and the airframe designer. The technique has its genesis rooted in military fighter applications, but is equally applicable to commercial aircraft/propulsion systems, helicopters, business aircraft, auxiliary power units, and other gas turbine installations.

As discussed previously, the Monte Carlo method was selected as the statistical tool because of its inherent ease of use and flexibility in handling a large number of variables with various distribution types that can be treated either as statistically independent or, if required, as having a statistical dependency.

2. REFERENCES

2.1 Applicable Documents

The following publications form a part of this document to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this document and references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

2.1.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or 724-776-4970 (outside USA), www.sae.org.

- | | |
|----------|--|
| ARP210 | Definition of Commonly Used Day Types (Atmospheric Ambient Temperature Characteristics Versus Pressure Altitude); Supersedes MIL-HDBK-310 and MIL-STD-210A |
| AS681 | Gas Turbine Engine Steady-State and Transient Performance Presentation for Digital Computer Programs |
| AIR1419 | Inlet Total-Pressure-Distortion Considerations for Gas-Turbine Engines |
| ARP1420 | Gas Turbine Engine Inlet Flow Distortion Guidelines |
| AIR5866 | An Assessment of Planar Waves |
| ARD50015 | A Current Assessment of the Inlet/Engine Temperature Distortion Problem |

2.1.2 ASME Publications

Available from ASME, 22 Law Drive, P.O. Box 2900, Fairfield, NJ 07007-2900, Tel: 973-882-1170, www.asme.org.

- | | |
|----------------------|--|
| ASME 97-GT-48 | Some Applications of the Monte Carlo Method to Gas Turbine Performance Simulations: Krurzke & Joachim |
| ASME 91-GT-402 | A Global Approach in Evaluating Inlet/Engine Compatibility: Csavina & Denney |
| Transactions of ASME | Journal of Engineering for Gas Turbines and Power, Aircraft Engine Control Mode Analysis, pp. 838-844: Brown & Elgin |

2.1.3 U.S. Government Publications

Available from the Document Automation and Production Service (DAPS), Building 4/D, 700 Robbins Avenue, Philadelphia, PA 19111-5094, Tel: 215-697-6257, <http://assist.daps.dla.mil/quicksearch/>.

- | | |
|--------------|--|
| MIL-HDBK-310 | Global Climatic Data for Developing Military Products (Superseded by SAE ARP210) |
| MIL-STD-210A | Global Climatic Data for Developing Military Products (Superseded by MIL-HDBK-310 then SAE ARP210) |

2.2 Applicable References

1. Rubinstein, R.Y., Simulation and the Monte Carlo Method, John Wiley and Sons, New York, 1981.
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7. Swan, Heads I Win, Tails You Lose, Dr. Dobb's Journal, December 1993.
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2.3 Symbols and Abbreviations

Alpha, alpha or AOA	Angle of Attack
ALT or alt	Altitude
APU	Auxiliary Power Unit
BLD	Compression system bleed
Beta, beta or AOSS	Angle of Sideslip
Cd	Nozzle Discharge Coefficient
CFD	Computational Fluid Dynamics
Cf	Nozzle Flow Coefficient
CIP	Component Improvement Program
CPU	Central Processing Unit
DIC	Circumferential Distortion Indices
DIRT	Tip Radial Distortion Indices
DIRH	Hub Distortion Indices
DPRA	Change in Available Pressure Ratio
DPRO	Change in operating line pressure ratio

DPRS	Change in stability pressure ratio
DSM	Change in stability margin
EFH	Engine Flight Hours
FOD	Foreign Object Damage
GETUP	Generalized Engine Throttle Usage Prediction
HPC	High Pressure Compressor
HPX	Horsepower Off-take
HR	Hours
Ks	Probability factor
LCG	Linear Congruential Generator
M or Mn	Mach Number
Max or MAX	Maximum
MC	Monte Carlo
MES	Main Engine Start
N	Weibull distribution coefficient
N/θ	Corrected Rotational Speed
NPR	Nozzle Pressure Ratio
PLA	Power Lever Angle
PR	Component Pressure Ratio
PRO	Operating line Pressure Ratio
PRS	Stability limit line Pressure Ratio
P^2/W	Combustor Loading Parameter
P/P	Component Pressure Ratio
RSS	Root Sum Square
REC	Inlet pressure recovery
SFC	Specific Fuel Consumption
SM	Stability Margin
Smavl or SM AVAIL	Available Stability Margin

SSA	Statistical Stability Assessment
TAC	Total Accumulated Cycles, where a cycle represents a defined sequence of engine throttle transients
TRANS	Transient
$\Delta T/T$	Temperature Ratio
$W\sqrt{\theta/\delta}$	Corrected Airflow
a	Weibull distribution coefficient
b	Weibull distribution coefficient
f/a	Fuel Air Ratio
g	Acceleration due to gravity
m	Probability distribution central moment, mean
s	Probability distribution central moment, standard deviation
Δ	Delta change
η	Efficiency
σ	Standard deviation
σ_{NEW}	Standard deviation of new engine performance
σ_{OH}	Standard deviation of overhauled engine performance

3. DESCRIPTION OF MONTE CARLO METHODOLOGY

Monte Carlo methodology is a modeling technique that is widely used throughout engineering, science, and commerce. It enables the overall statistical variability of a process to be determined, based on the known variability of the process inputs. Statistical distributions are used to define the variability of the inputs to the process, and these are sampled randomly many times to provide information about the overall process output. The input parameters to the process are defined at a level such that their distribution can be readily defined or empirically determined. These distributions can take many forms, ranging from uniform or normal distributions to non-uniform discrete distributions that capture the true variation of an input parameter. To create a statistically significant database, the method requires that the individual inputs are randomly sampled a sufficiently large number of times. To achieve this, a computer simulation of the process is generated and a Random Number Generator used to sample each input distribution many times. The complexity of the simulation depends on the process being modeled. The flexibility of the technique is such that the simulation can be extended to large systems and easily tailored to include input parameters that are statistically interdependent as well as those that are statistically independent. The technique facilitates parametric analyses within the model, establishing output sensitivities due to changes in component parameters.

A simple example of a Monte Carlo simulation is shown in Figure 1. A process is represented wherein two washers, from different sources, are to be fitted to a unit and an understanding of the total variation in thickness is required. Washer A is produced from a process that yields much greater variability than that for washer B. The cumulative probability of the thickness for each washer is shown along with its probability density function and this is sampled repeatedly using uniformly-distributed random numbers. The overall thickness of each combination of washers is then calculated. By considering a sufficiently large number of samples, the resulting distribution of combined washer thickness is derived.

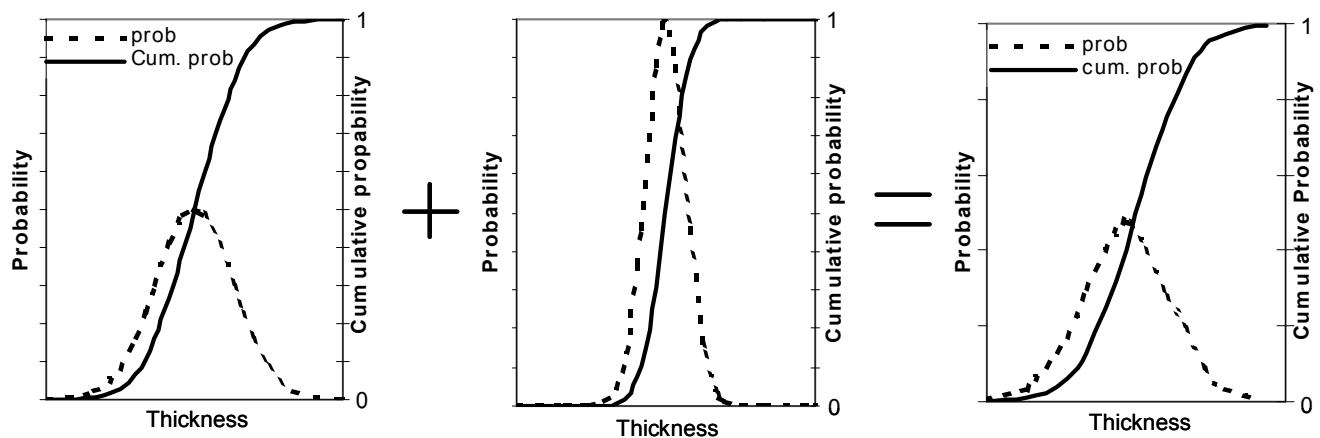


FIGURE 1 - EXAMPLE OF MONTE CARLO SIMULATION

In this example, simple normal distributions were assumed for two independent variables.

The Monte Carlo simulation is applied generally to very much more complex problems where there are many variables, each with their own statistical distribution, the variables being either statistically independent or statistically dependent. Where the value of a variable is dependent on the value of another variable, their values can be selected using the same random number or there may be a dependency such that a given value of a variable will lead to a new constraint on another variable. Such relationships between variables can be programmed into the Monte Carlo simulation.

For complex processes, the logical sequence of events or components of the process need to be determined and used to develop the Monte Carlo simulation. The simulation will then interrogate the variable distributions in turn and [should] reflect the actual process. An example process is shown in Figure 2. In this example, the process output, D, is a function of variables A, B and C. A and B are statistically independent while the value of C is a function of B and has a statistical distribution. The simulation is required to interrogate model C with a value of B and a random number in order to obtain a value of C.

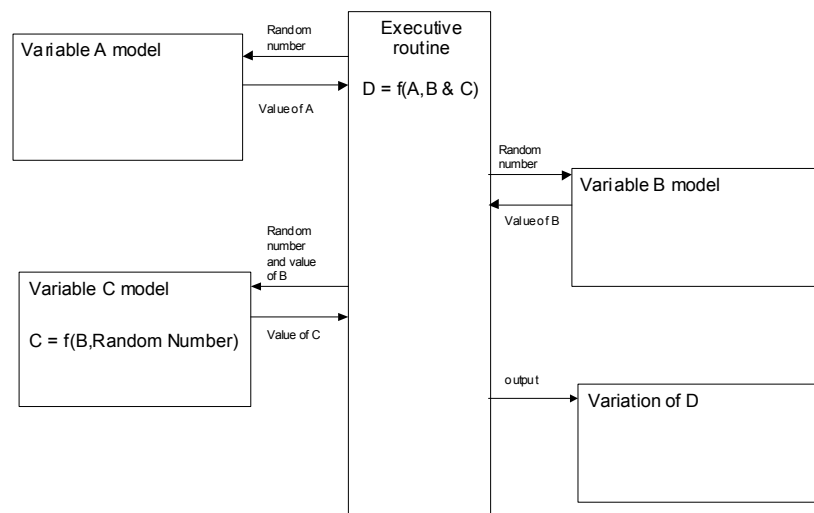


FIGURE 2 - EXAMPLE OF MONTE CARLO SIMULATION PROCESS

The routine that forms the basis of the simulation has been termed the 'executive routine.' Additional functionality can be built into this routine to give the simulation flexibility. For example, being able to assign given variables a fixed value enables sensitivity studies of specific variables to be performed. It also enables parametric studies to be carried out.

Two important considerations when developing a Monte Carlo simulation are the sample size and the choice of random number generator. The appropriate sample size is usually optimized during the development of the model. A sufficiently large number of samples is required to ensure adequate repeatability of the output but with a manageable simulation run time, as discussed in 6.2.2. The random number generator must be capable of producing a distribution of numbers appropriate to the sample size being considered. A rule of thumb is that the period of the generator be at least twice the size of the number of samples required by the application. Some random number generators enable the user to specify a seed such that a repeatable series of random numbers is generated. This feature is useful for debugging the application or for performing some sensitivity studies. These issues are discussed further in Appendix A.

The flexibility of the Monte Carlo simulation makes it an ideal technique for assessing complex statistical problems such as gas-turbine-engine stability-margin assessment. It should be noted that the accuracy of the simulation output depends on how well the process elements and their distributions are modeled.

4. STATISTICAL STABILITY ASSESSMENT (SSA)

The factors that are considered in a stability assessment are shown in Figure 3 (although this is not an exhaustive list). Historically, a simple arithmetic summation of all factors was performed to arrive at the required compression-system stability margin at a given flight condition. This historic practice was defined as the 'worst case' stability margin stack-up. Subsequently, as defined in Appendix A of AIR1419, it was recognized that the probability of all worst-case values occurring simultaneously was low and that some of the factors should be summed statistically. The factors defined as being random in nature included items such as engine-to-engine variation. These were assumed to have normal distributions and their effects on stall margin were summed using a root-sum-square method. This form of stability assessment has become the recognized current practice used throughout industry. While having a statistical element, the methodology does not recognize factors with non-uniform distributions or provide insight into the probability of instability in service. Further discussion of this practice is presented in Appendix B.

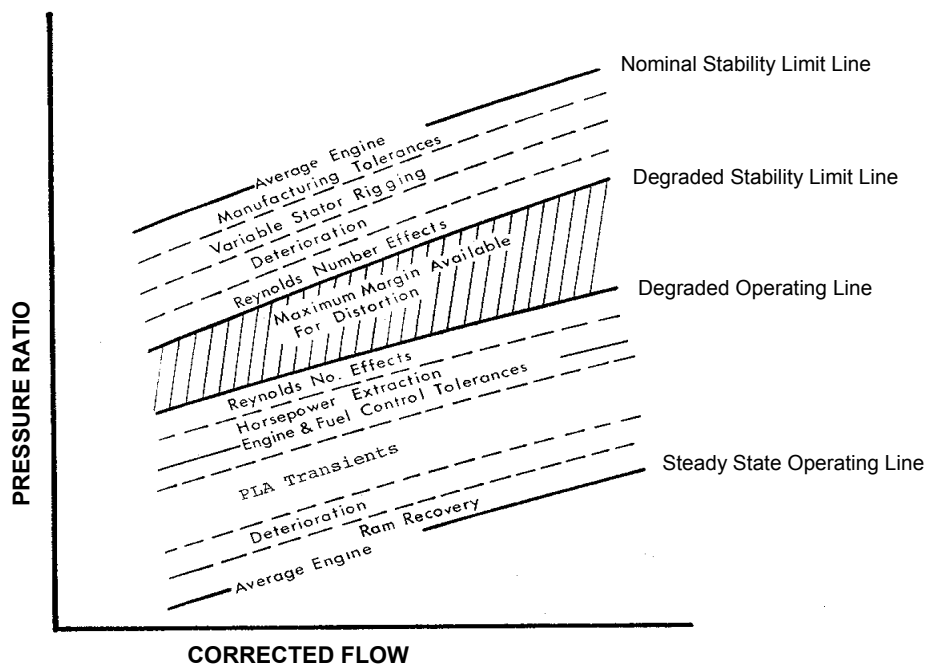


FIGURE 3 - TYPICAL COMPRESSOR DESTABILIZING FACTORS

In addition, the increasing complexity of engine control systems and variable geometry are such that it becomes difficult to select a small number of worst-case scenarios that, if shown to have positive residual stability margin, would indicate the engine to be stall-free. The statistical stability assessment methodology has been developed as an extension to the current stability audit to enable parameters with non-normal distributions to be appropriately considered and to give insight into stability-margin usage and potential stall rate. The use of the Monte Carlo technique enables very complex installations and processes to be modeled allowing the simulation to be used as a design tool to identify critical flight conditions.

It is important to note that the existing stability assessment method of AIR1419 can be a sub-set of the SSA methodology. This can be accomplished by including a default option in the SSA model that duplicates the existing stability assessment.

The statistical assessment requires the physics underlying the operational conditions and the vehicle characteristics to be input with the variability and distributions that reflect the system usage. The methodology accomplishes this by modeling the significant elements that influence the stability of the compression-system components, incorporating the independent and dependent variability of the elements, as appropriate.

There are many factors that influence the stability of the compression system of the engine. For the purpose of simulating operation in an aircraft, they can be divided into four major groups. These groups are the environment, the physical installation, the usage, i.e. the operational condition, and the engine. These groupings are thought to be fairly generic, but can be tailored or changed for different gas-turbine applications, as necessary. The Statistical Stability Assessment requires a model for each of these groups to be produced such that, when interrogated, they will simulate the overall operational experience.

4.1 System Architecture

The basic architecture for a Monte Carlo simulation was described in Section 3. The architecture required for a statistical stability margin assessment is shown in Figure 4. The executive routine interrogates the models in the logical order to define a given flight condition. First, the environmental model is interrogated to set the ambient conditions. Second, the usage model provides specific data such as altitude, Mach number, aircraft attitude, and power lever angle (PLA) position. Then, these data are used as input into the aircraft model to determine engine-entry-flow parameters. Having determined the relevant data needed to describe a flight condition, an engine model is used to assess stability margin usage. The engine model combines the effects of the external influences with the internal engine influences.

4.2 Executive Routine

The executive routine serves to control the flow of data to and from the models, and controls the interrelationships between parameters. The executive routine produces a random number for each variable using a suitable random number generator. There are several types of random number generators. Important considerations when choosing a generator are discussed in Appendix A. The random number is supplied to the model along with the relevant dependency data. For example, a call for an angle of sideslip will require not only a random number, but also details of Mach Number, altitude, and angle of attack. Based on the information supplied, each model returns a parameter value from its associated probability distribution. This process continues until a complete definition of a flight point is produced and a stability margin usage obtained. In practice each model will be utilized several times during each pass through the simulation. This is because some variables will be dependent on others from the same model. There are also some cases when the simulation may need to address the models out of the simple logical sequence or even perform a number of iterations. An example of this would be the calculation of intake pressure recovery and distortion that are dependent on the engine flow. The executive routine must be organized to manage such cases efficiently and minimize computer time. More examples of parameter dependencies are provided in Appendix C.

The executive routine can accommodate varying degrees of complexity. It can be a simple simulation of a particular problem or it can be a complex interactive computer program that provides the user with many options for tailoring the analysis. In its most complex form, it should be possible to assign a fixed value to any variable, to change parameter dependencies, and to offer options for different outputs. The more flexible the simulation, the more useful it will become for enabling parametric trade studies to be performed and "what-if" type assessments to be made. An executive routine could be programmed fairly simply in any computer language. Alternatively, there are commercially available codes that can serve as the executive routine by integrating the various models and performing the Monte Carlo function.

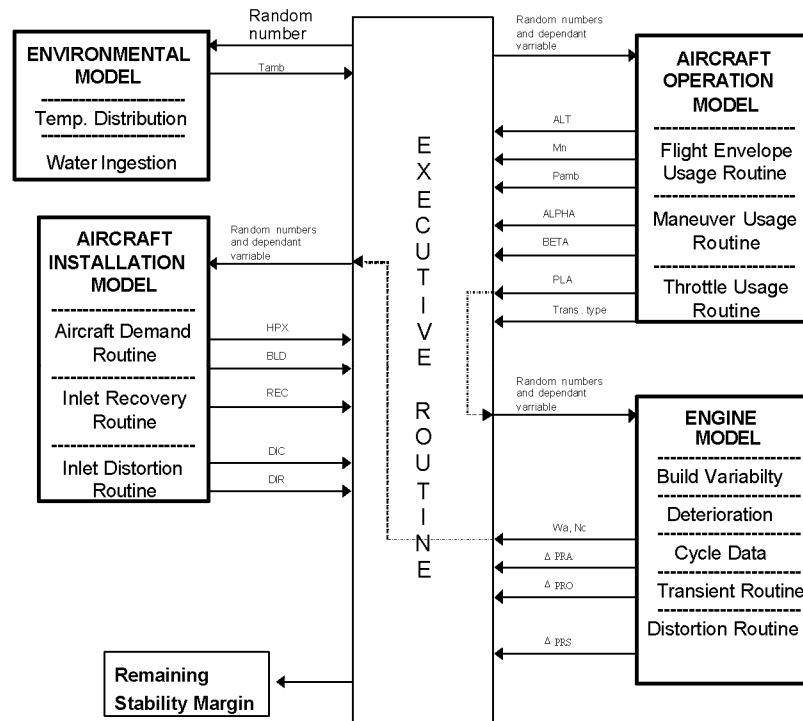


FIGURE 4 - AIRCRAFT/ENGINE SYSTEM ARCHITECTURE

4.3 Simulation Models

For the purpose of simulating a gas turbine engine in an aircraft, four basic groups of data have been identified. It is recommended that each group be considered as a particular model, containing a number of variables. As with the executive routine, the complexity with which variables are modeled depends on the use of the simulation, but the better the data, the more meaningful the simulation will be.

The four models are:

- Environmental model to define the variability of the atmospheric conditions. This model should contain parameters such as temperature, pressure, and humidity.
- Usage model to define the operation of the aircraft including capabilities, missions, and pilot operating characteristics. The model is used to determine values of altitude, Mach number, AOA, AOSS, power lever angle and engine transient type.
- Aircraft model to define values of inlet recovery, distortion, horsepower off-take, and customer bleed. This model could be extended to consider variability of these items.
- Engine model that defines the engine physical and functional characteristics such as, build tolerances, deterioration, and other variables, and how the engine operates in or responds to the given flight condition. This model should be capable of providing stability margin used and stability margin remaining back to the executive routine to form part of the output data. Both steady-state and engine transient simulations are used in this model.

Greater detail of the data that should be included in each model is contained in Section 5.

An appropriate statistical distribution needs to be defined for the applicable variables contained within the models. The normal distribution is an obvious choice for modeling variables with a truly random element, such as build scatter, while a discrete distribution is more appropriate for variables such as flight-envelope usage or service demands. A brief description of some useful distributions for the modeling of factors that influence engine stability margin is given in Appendix D.

4.4 Model Execution

The modeling of a complex system, such as an aircraft and its propulsion system, can involve a great number of variables. When performing the Monte Carlo simulation it is important that a sufficient number of passes be considered. During the development of a Monte Carlo simulation model, cases should run with a range of passes to find the optimum number of passes required for convergence. During this exercise, a careful watch should be kept on the random number generator as it may begin to repeat after a given number of passes and hence, invalidate the randomness of the simulation. Each pass through the simulation is considered as a sample. The user should be very careful to avoid the pitfall of using too few passes when the solution would not converge to an acceptable accuracy. Guidelines for ensuring that this is avoided are provided in 6.2.2.

5. DESCRIPTION OF SIMULATION MODELS

This section gives examples of how the individual models can be constructed. Although the following models and their subroutines are representative of a modern-day fighter application, they are indicative of what would be required to simulate any gas turbine application.

5.1 Environmental Model

The environmental model, shown in Figure 5, describes the ambient air temperature (at altitude) and other weather conditions, which are to be considered for the flight envelope defined by the usage model. This model depicts the demands on the engine due to environmental conditions. The most obvious is ambient temperature, as it affects the corrected rotor speeds and airflow and, thereby, the fan and compressor match points. Other factors can be included such as water ingestion from heavy rainfall. Statistical data on worldwide weather conditions are readily available from a number of sources. A typical example is MIL-HDBK-310. Such environmental models may be easily incorporated into a Monte Carlo simulation. Engine synthesis models already accept ambient temperature as an input variable, thereby thermodynamically responding to temperature variations. These temperature variations affect clean available stability margin due to fan and compressor match point changes.

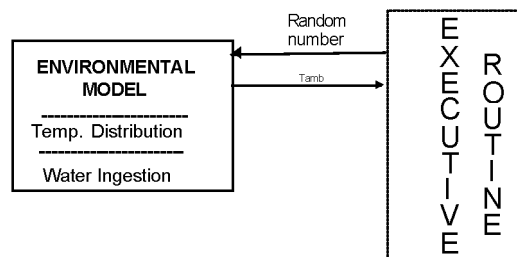


FIGURE 5 - ENVIRONMENTAL MODEL

If results are desired for variable temperature as opposed to using specific “days” (i.e., hot, cold, std), a variable temperature distribution can be found in MIL-HDBK-310, which gives the cumulative probability of temperature variation relative to Standard Day temperatures. This distribution gives more extreme temperatures than the cold and hot day atmospheres, but the probabilities are very low for these extremes.

Theater-specific environmental models can also be produced. In this case, environmental data can be obtained from the closest weather recording post. An example of a theatre-specific ambient temperature distribution is shown in Figure 6, compared to that for a standard day. It shows there is a significant shift (approximately 35 °F cooler) in the distribution for Eielson AFB Alaska, which would keep corrected flows higher at their scheduled military/maximum power settings, thereby generating maximum distortion levels at a much higher probability than for a standard atmosphere. Fan and compressor rotor tip clearances would also be affected especially at take-off conditions, where rotors would be colder because they have yet to be thermally stabilized at power.

A relevant rare-condition-driven stall event (although important) would be one destabilized by extreme water ingestion. In this case, the cycle deck must have inputs for water to air ratio and be able to respond (thermodynamically and aerodynamically) to show the impact on both fan and compressor stability margins. The ability to accurately model rain ingestion is a significant challenge for cycle models. Theater-specific statistical weather data can be found by referring to the FAA weather office or the National Climatic Data Center. In the case of the U.S. Air Force, a central information source is available at the Wright-Patterson Air Force Base Weather Office.

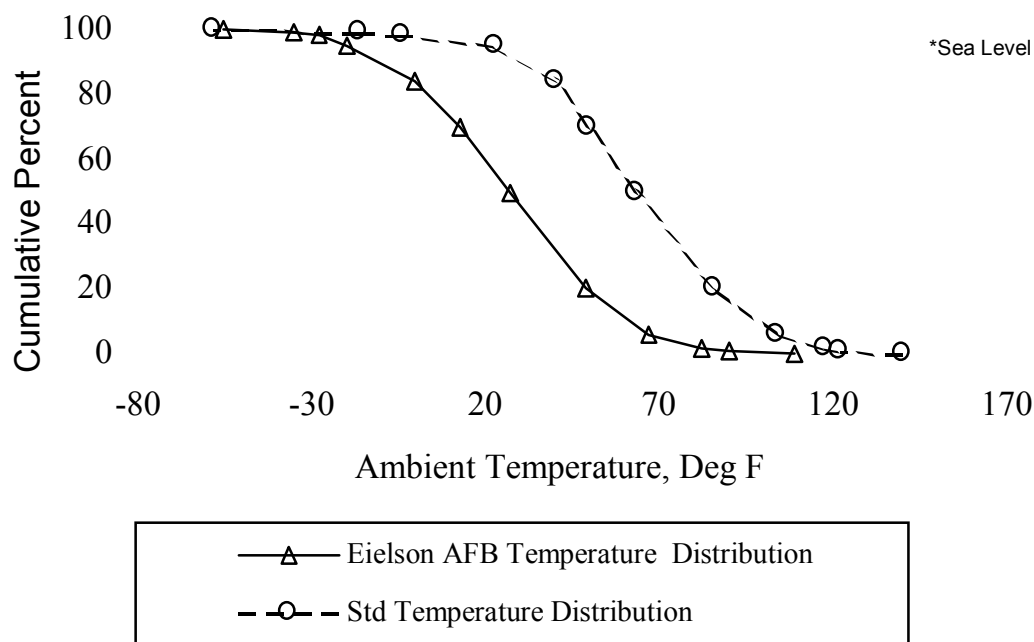


FIGURE 6 - THEATRE-SPECIFIC AMBIENT TEMPERATURE DISTRIBUTION

5.2 Aircraft Operations Model

The Aircraft Operations Model describes the mission or operational conditions to be analyzed in terms of altitude, Mach number, angle of attack and sideslip, engine throttle position (PLA), and type of throttle transient. An aircraft operations model diagram is shown in Figure 7.

For the aircraft operations model, which depicts where and how the aircraft is flown, historical data from flight recorders, and flight simulator data flown to representative missions, if available, are most appropriate for this application. However, in some cases estimated data are required.

The aircraft operations model consists of three routines that are discussed from a fighter aircraft perspective. They are the Flight Envelope Usage Routine, the Maneuver Envelope Usage Routine, and the Throttle Usage Routine.

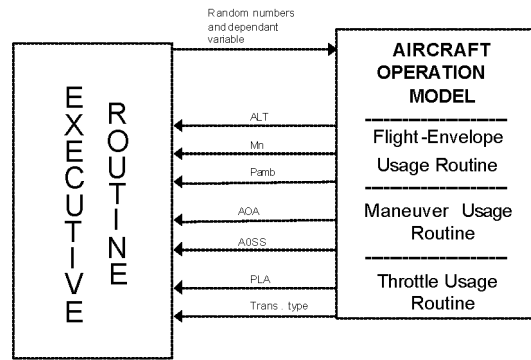


FIGURE 7 - AIRCRAFT OPERATIONS MODEL DIAGRAM

5.2.1 Flight-Envelope Usage Routine

Since this section on flight-envelope usage is one of the first discussions of the technique for development of probability distributions to be used in this overall Monte Carlo simulation, greater detail will be presented in this section than in the sections that follow.

Mach/altitude usage data from onboard recorders from a fleet of modern military fighters are shown in Figure 8. The time in hours per 1000 hours and the number of transfers per 1000 hours into and out of each Mach/altitude block are shown. The data were not available in more detail than is shown, and it is assumed the usage is distributed uniformly within each block.

It should be noted that the number of blocks should be chosen such that there is not a significant variation of residence time across a block. While a large block was used to cover the top left-hand corner of the flight envelope, it should be noted that this would result in overexposing the engine to areas of minimal stall margin. A more representative approach may be to represent this region with a number of smaller blocks that would more accurately represent actual usage.

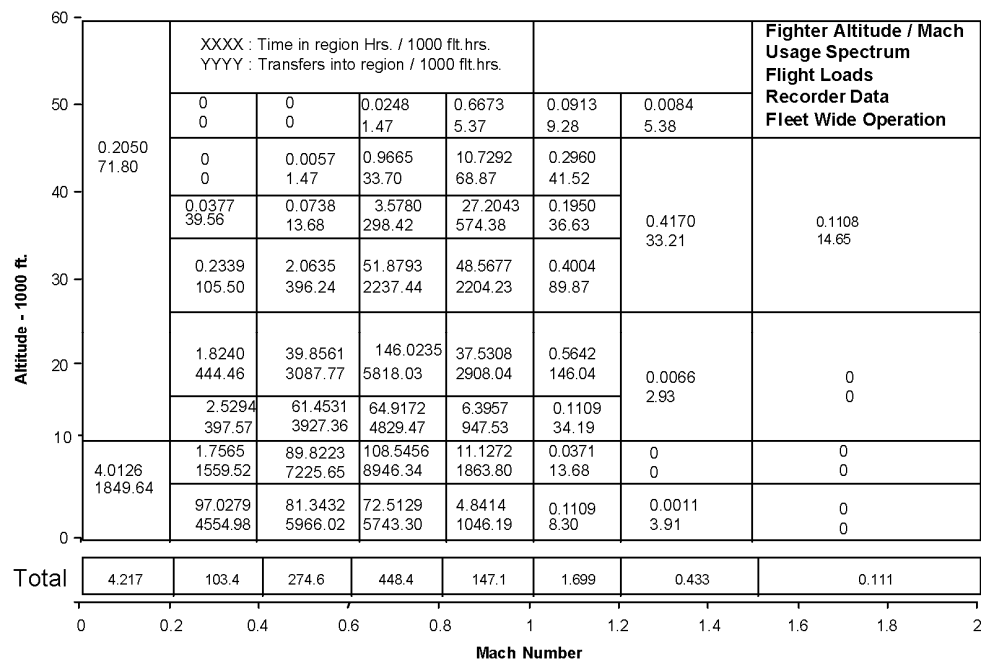


FIGURE 8 - TYPICAL FLIGHT-ENVELOPE USAGE DATA

In order to develop a statistical model, the data must be characterized as a function of both altitude and Mach number. The Mach number distribution will be addressed first.

The summation of time in each Mach number block is shown at the bottom of Figure 8. The data from this summation were converted to percentages of total time and used to construct the discrete probability distribution as a function of Mach number, as shown in Figure 9. This approach is consistent with the assumption that the data are uniform within the original blocks. These same data were then converted to the equivalent cumulative probability distribution shown in Figure 10. The cumulative probability distribution is the form of the data that is used in the Monte Carlo simulation. A random number between 0% and 100% is used to obtain a Mach number from the cumulative probability distribution shown in Figure 10; this Mach number is then used for a given pass through the overall simulation. Since the random numbers are uniformly distributed, the derived Mach numbers for various passes through the simulation will be distributed as in the original curve.

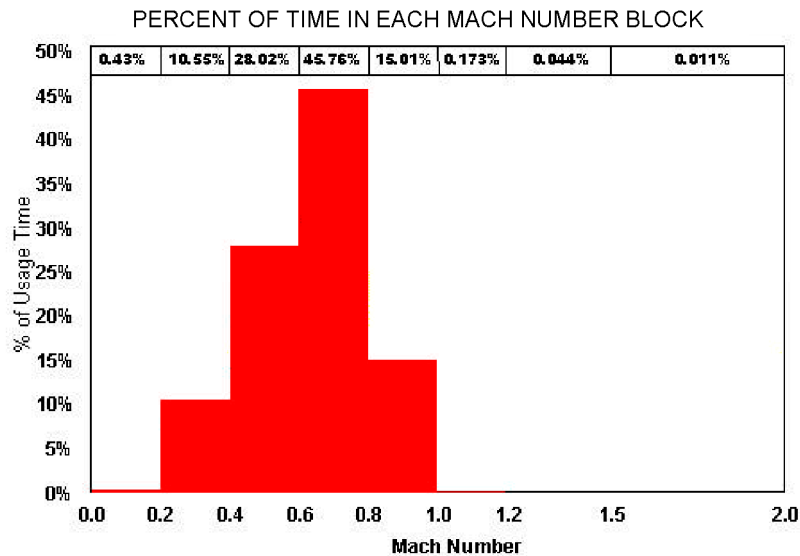


FIGURE 9 - MACH NUMBER PROBABILITY DISTRIBUTION

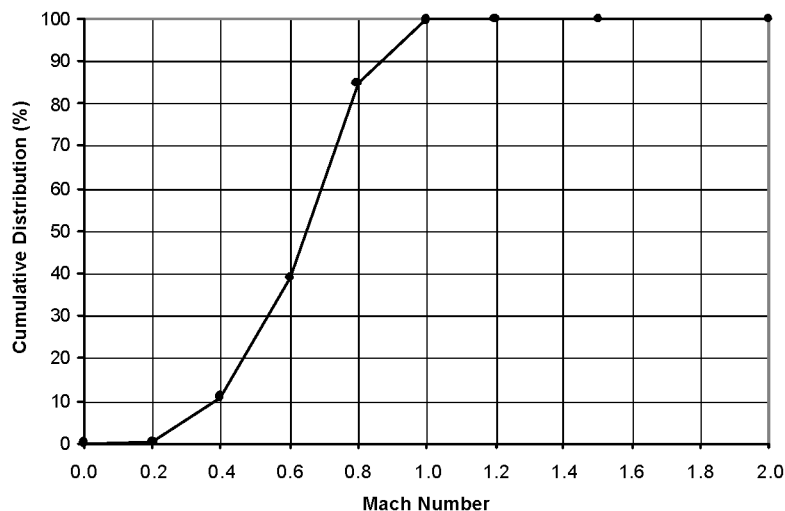


FIGURE 10 - MACH NUMBER CUMULATIVE PROBABILITY DISTRIBUTION

To demonstrate that the original data are reproduced to a high degree of fidelity, the data of the cumulative distribution (Figure 10) were used in a Monte Carlo simulation and the resulting Mach number probability distribution is compared to the original distribution in Figure 11.

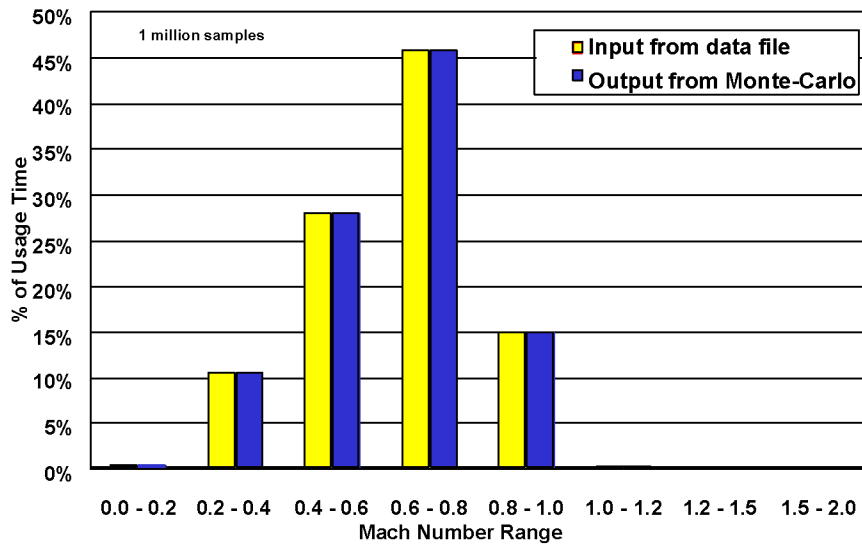


FIGURE 11 - COMPARISON OF MONTE CARLO SIMULATION DATA

The altitude distributions for each of the Mach number blocks were obtained in a similar manner. It was desired to break the altitudes into increments of 5000 feet. In several cases, the original data are for blocks with a larger altitude range. Again, it was assumed that the data were distributed uniformly within the original blocks and the distributions were constructed accordingly. The resulting cumulative probability distributions for each of the Mach number blocks are presented in Figure 12.

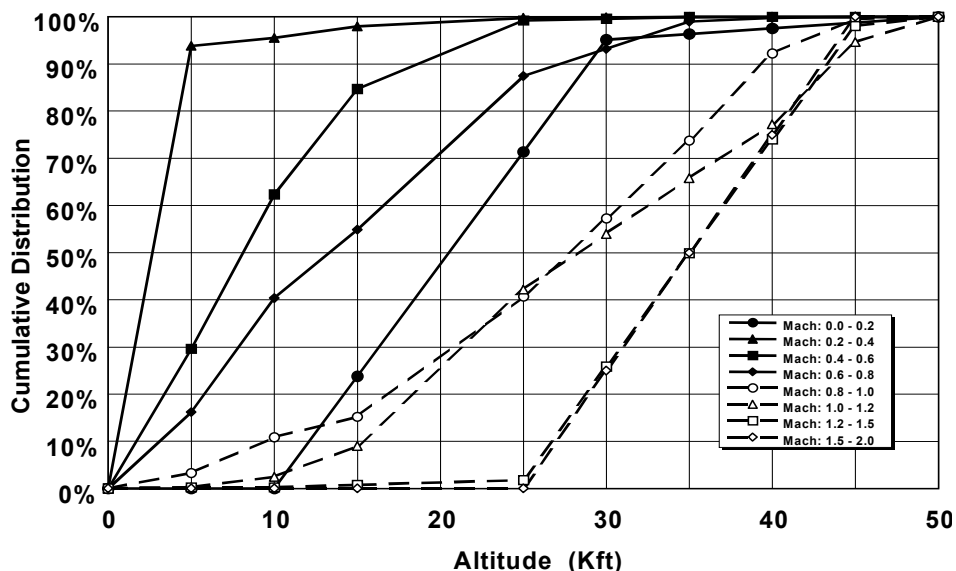


FIGURE 12 - ALTITUDE CUMULATIVE PROBABILITY DISTRIBUTIONS

Again, to demonstrate that the original distribution is reproduced to a high degree of fidelity when used in a Monte Carlo simulation, the resulting probability distribution is compared to the original distribution (for one Mach number block) in Figure 13.

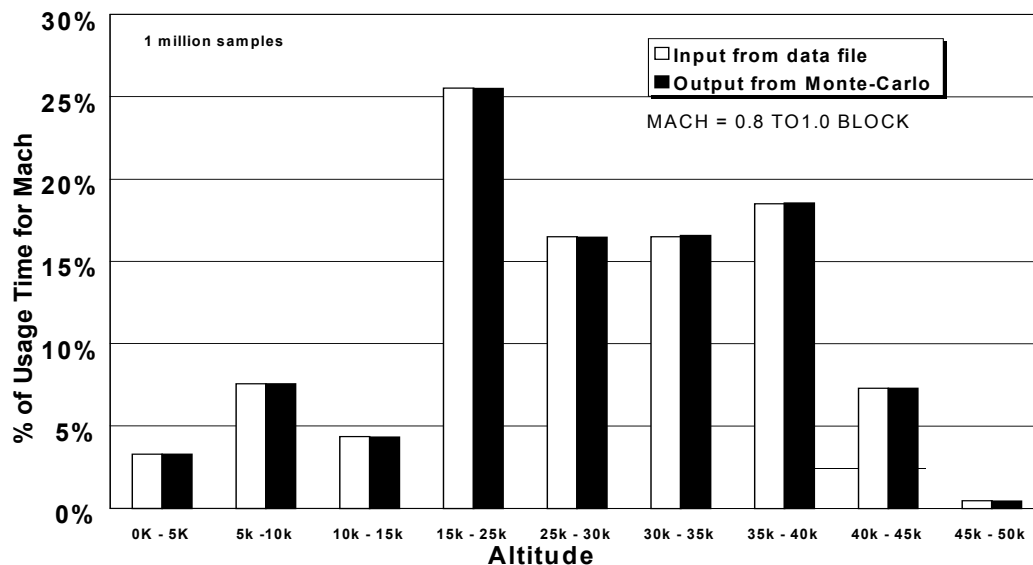


FIGURE 13 - COMPARISON OF MONTE CARLO SIMULATION DATA

Using the statistical usage data and two random numbers, a Mach number and altitude can be generated for use in subsequent steps of the Monte Carlo simulation.

5.2.2 Maneuver Envelope Usage Routine

Angle of attack and sideslip conditions are usually restricted within the aircraft design maneuver limits for a given Mach number. A representative maneuver envelope is shown in Figure 14. The frequency of occurrence of each AOA/AOSS condition is distributed within the maneuver limits. Thus, the distribution is truncated at the AOA/AOSS design limits. Aircraft loads data from flight recorders or simulator outputs can also be a source to obtain the AOA/AOSS distributions. Departed flight may also be modeled, if desired. In this example, the frequency of occurrence of each AOA/AOSS condition is assumed to follow normal distributions truncated at the maneuver limits. Thus, both AOA and AOSS are dependent on Mach number, and AOSS is also dependent on AOA.

Another consideration that should be included in the AOA/AOSS selection is the limitation on angle of attack and sideslip imposed by maximum "g" capability of the aircraft. Within the ranges of these values that are considered here, limiting AOA and AOSS are inversely proportional to the dynamic pressure. For example, if at a given Mach number, the angle of attack is limited to 30 degrees at 30kft altitude, then the maximum AOA is limited to 10 degrees at a lower altitude of 3200 feet where the dynamic pressure is three times the value at 30kft.

At this point Mach number, altitude, AOA, and AOSS have been determined for use in subsequent steps of the simulation.

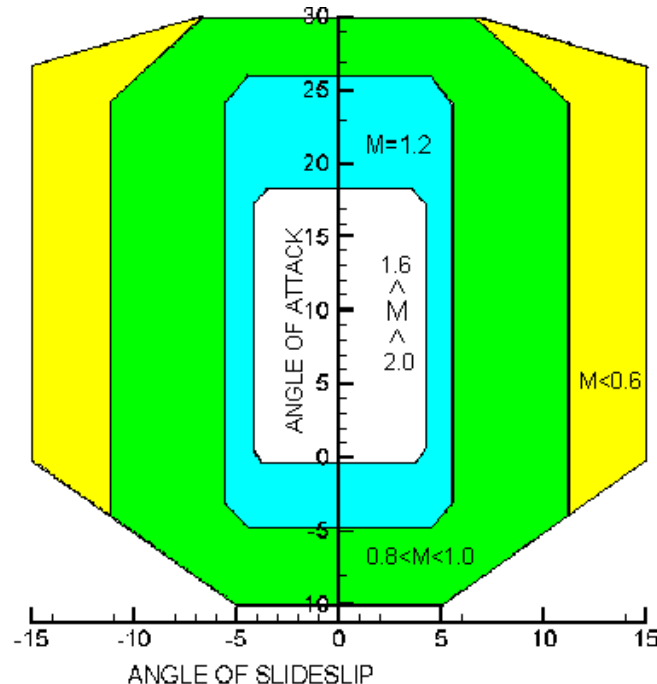


FIGURE 14 - MANEUVER ENVELOPE USAGE MODEL DIAGRAM

5.2.3 Throttle Usage Routine

Aircraft engine throttle usage data may be used to identify the types of destabilizing throttle transients that are typically performed, and to determine their frequency of occurrence and where in the flight envelope (altitude, Mach) they are encountered.

As an example, analysis of mission data gathered on a twin-engine fighter aircraft evidenced the following types of destabilizing transients:

- High power accelerations (rapid throttle movement from low or part power to near intermediate or above)
- Low power accelerations (from near idle to some higher power)
- Bode (from intermediate or above to part power and return in rapid succession)
- Deceleration (chop from near intermediate to near idle)
- Afterburner light
- Afterburner cancellation

In addition, fixed-throttle time and ground time prior to takeoff were taken into account. These transient types were chosen in order to capture known destabilizing effects. High and low power accelerations, Bodes, and afterburner cancellations are known to challenge the compressor, and decelerations and A/B lights are known to challenge the fan. Fixed-throttle operation may occur during an aircraft maneuver that generates high inlet distortion that may challenge either the fan or the compressor.

For analysis purposes, the throttle usage data was organized into two parts: (1) frequency of occurrence for each throttle transient type, and (2) the likelihood of encountering a particular throttle transient at any given altitude/Mach condition. In Table 1, the following frequencies were observed:

TABLE 1

Transient Type	Description	Occurrences per flight hour
1	High power accel	46.61(46 610/1000 EFH)
2	Low power accel	28.49(28 490/1000 EFH)
3	Bode	0.82(820/1000 EFH)
4	Decel	2.92(2 920/1000 EFH)
5a (5b)	A/B Light (cancellation)	18.17(18 170/1000 EFH)
6	Fixed-throttle	73.08(73 080/1000 EFH)

For this example, the above throttle transient types showing the percentage usage in various altitude/Mach bins are presented in Figure 15. The numbers in each block represent the percentage likelihood of encountering any particular transient types that are initiated in that altitude/Mach region. Similar outputs may be made for the other transient types.

Probability of Throttle Transient Type as a Function of Mach and Altitude

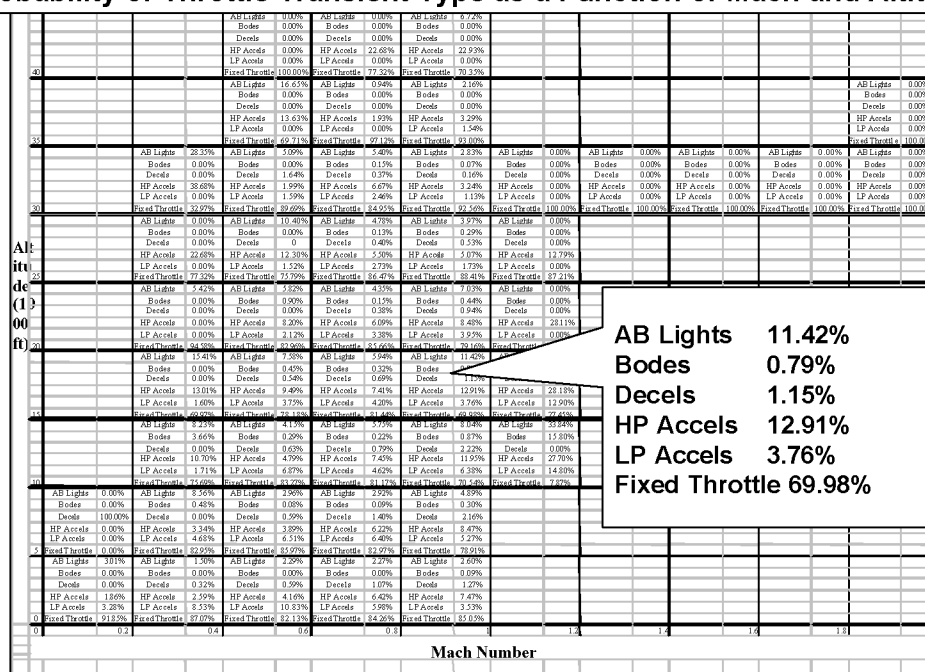


FIGURE 15 - TRANSIENT TYPE OCCURRENCES AS A FUNCTION OF ALTITUDE AND MACH NUMBER