



AEROSPACE INFORMATION REPORT

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Measurement Uncertainty and Consumer Risk

RATIONALE

AIR6553 has been reaffirmed to comply with the SAE Five-Year Review policy.

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1. SCOPE

This document addresses measurement uncertainty and consumer risk as they relate to AS8879 thread inspection. It describes the rationale, theory and methodology used to generate the technical content of the AS5870. The document describes how to calculate measurement consumer risk. It documents all of the calculation methods which industry employs today to calculate what is commonly called measurement uncertainty (Appendices A, B, C, D, E and F). These, in turn, are used to calculate measurement uncertainty ratios which are required inputs to calculate measurement consumer risk.

Users of this document can apply the information described herein for the evaluation of the capability of their measurements based on the measurement consumer risk. It involves the analysis of the measurement (product) distribution and biases of both the product and measurement system distributions. It protects the consumer from the worst case distribution results.

2. REFERENCES

2.1 Applicable Documents:

The following publications form a part of this standard to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this standard and references cited herein, the text of this standard takes precedence. Nothing in this standard, however, supersedes applicable laws and regulations unless a specific exemption has been obtained

2.1.1 SAE Publications

Available from SAE, 400 Commonwealth Drive, Warrendale, PA 15096-0001.

AS5870 Thread Inspection Practices

AS8879 Screw Threads, UNJ Profile, Inch, Controlled Radius Root With Increased Minor Diameter

2.1.2 ASME Publications

Available from the American Society of Mechanical Engineers (ASME) through the web at <http://www.asme.org> or in writing at ASME, 22 Law Drive, Box 2900, Fairfield, NJ 07007-2900

ASME B1.2 Gages and Gaging for Unified Inch Screw Threads

ASME B46.1 Surface Texture (Surface Roughness, Waviness and Lay)

2.1.3 FAA Publications

Available from Federal Aviation Administration, 800 Independence Avenue, SW. Washington, DC 20591

REPORT NO: FAA-IR-01-02 - AVIATION FASTENER AUDIT FINAL REPORT JUNE 13, 2001

Available at: www.faa.gov/library/reports/media/FASTENER_AUDIT_REPORT.pdf

3. DEFINITIONS

3.1 Average Outgoing Quality Limit (AOQL)

Maximum Average Outgoing Quality over all possible values of incoming product quality level for a given acceptance sampling plan.

3.2 Average Outgoing Quality (AOQ)

The average quality of outgoing product after sampling inspection for a given steady value of incoming product quality.

3.3 Calibration

Set of operations that establish, under specified conditions, the relationship between values of quantities indicated by a measuring instrument or measuring system, or values represented by material measure or a reference material, and the corresponding values realized by standards. [VIM, 6.11]

3.4 Certification

The act of designating that M&TE has been calibrated and are expected to perform within established performance specifications for the duration of the assigned calibration interval with a historical quality level greater than or equal to a quality level target or falling within a defined quality corridor.

3.5 Confidence Interval (CI)

The interval of a measurement result that is estimated to contain an associated percentage, as represented by the confidence level, of the distribution of possible measurement results, e.g., ± 0.0002 inch.

3.6 Confidence Level (CL)

The probability that a measurement result made at the time of its evaluation is within its associated confidence interval.

3.7 Consumer Risk (CR) (single measurand)

A numerical estimate of the probability that the true value of a measurand (for example: pitch diameter at 68 deg F) falls outside a given performance specification, but is measured to be within the guardband limit.

3.8 Engineering Tolerance

The total amount by which a specified measurand is permitted by specification to vary.

3.9 Error Ratio (ER)

The ratio between the Maximum Allowable Error (also referred as Engineering Tolerance) and the Measurement System Total Error for a given measurement on the same confidence level basis.

3.10 Expanded Error

The sum of the measurand bias and extended uncertainty that bound the values of expected error for a given confidence level.

3.11 Expanded Uncertainty

The lower and/or upper confidence intervals that bound the expected values of a random variable for a given confidence level.

3.12 Extended Uncertainty

The lower and/or upper confidence intervals that bound the expected values of a random variable for a given confidence level.

3.13 Guardband Factor (GBF)

A value between zero and one yielding the Guardband Limit when multiplied by the product tolerance

3.14 Guardband Limit (GBL)

Reduced acceptance limit that maintains the target measurement consumer risk

Note 1: Guardband limit, as defined here, is risk-based and is therefore consistent with the general basis of this standard.

Note 2: These limits are obtained by multiplying the Guardband Factor (GBF) times the Product Tolerance.

3.15 Historical Measurand Estimate Uncertainty (HMEU)

The confidence interval and associated confidence level of the measurand estimate based on its history. This value is required to apply guardband limits based on uncertainty ratios for metrology.

3.16 Historical Quality Level (HQL) (single measurand)

A fractional number expressed as a percentage, equal to the number of certifications since the last interval adjustment (for a single measurand from the same population) for a given item or class of Measurement and Test Equipment (M&TE), divided by the number of certifications during that interval, in which no values upon re-certification were observed to be outside the certification specification limits.

3.17 Measurand

Particular quantity subject to measurement, i.e., pitch diameter of a given thread plug gage at a known temperature. [VIM, 2.6]

3.18 Measurement and Test Equipment (M&TE)

All of the measuring instruments, measurement standards, reference materials, process control equipment and other equipment that are required to carry out a measurement. This term includes measuring equipment used in the course of manufacturing, testing and inspection, as well as that used in calibration.

3.19 M&TE Performance Specification Tolerance

The assigned measurement capability of the M&TE used to make the measurement.

3.20 Measurement Capability

The performance of a measurement system as intended to define, realize, conserve or reproduce one or more of its obtained measurement values.

3.21 M&TE Performance Specification Tolerance

The assigned measurement capability of the M&TE used to make the measurement.

3.22 Measurement System Expanded Uncertainty

The product of a coverage factor and standard uncertainty or combined standard uncertainty of the measurement system defining a confidence interval about the result of a measurement that is estimated to contain an associated percentage (as represented by the confidence level) of the distribution of possible measurement results.

3.23 Measurement System Total Error (MSTE)

The sum of the measurement system's systematic errors and the random error, with the random error evaluated at a specific confidence level.

3.24 Performance Specification

Specifications which describe the parameters and range to be calibrated, and the limits of allowable error or uncertainties considered acceptable. It is also referred to as the M&TE Performance Specification Tolerance, Measurement System Total Error and Measurement System Expanded Uncertainty.

3.25 Producer Risk (PR)

A numerical estimate of the probability that the true value of a measurand (for example: pitch diameter at a known temperature) falls inside a given performance specification, but is measured to be outside the guardband limit.

3.26 Quality Metric

Parameter used to define a sampling requirement, e.g., AOQL.

3.27 Rejectable Quality Level (RQL)

The minimum percent defective at which a lot will have 5 percent or less chance of being accepted when using a sampling plan or another statistically based monitoring system.

3.28 Sigma Model

The performance specification divided by the number of standard deviations in the product distribution.

3.29 Thread Groove Diameter

The diameter of an imaginary coaxial cylinder which cuts the thread profile in such a way to make only the thread groove equal to half the basic pitch. It is often referred to as "best wire size" pitch diameter. The measured thread groove diameter is accepted as an estimate of the pitch diameter when form variations are within specification limits.

3.30 Thread Pitch Diameter

The diameter of an imaginary coaxial cylinder which cuts the thread profile in such a way to make the thread groove equal to the thread ridge.

3.31 Traceability

Property of the result of a measurement or the value of a standard whereby it can be related to stated references, usually national or international standards, through an unbroken chain of comparisons all having stated uncertainties. The concept is often expressed by the adjective "traceable". [VIM]

3.32 Uncertainty (of measurement)

Parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand. [GUM]

3.33 Uncertainty Ratio (UR)

The Historical Measurand Estimate Uncertainty of the product divided by the Measurement System Expanded Uncertainty for a given measurement on the same confidence level basis (for example, both 1 sigma, 2 sigma, etc.).

4. DEFINITION EXPANSIONS

The following explains the differences of the GUM definitions as compared to how the terms are used in this document. The definition expansion numbers refer to the definition numbers above, prefixed by E to indicate they are expansions.

E3.5 Confidence Interval

Although this definition is not identical to the version of the GUM, its content is at a more appropriate technical level than the GUM definition, which is unnecessarily complex for the purposes of this AIR.

E3.6 Confidence Level

Although this definition is not identical to the version of the GUM, its content is at a more appropriate technical level than the GUM definition, which is unnecessarily complex for the purposes of this AIR.

E3.9 Error Ratio

In Metrology, the numerator of a relative capability ratio, such as error ratio or uncertainty ratio, is traditionally interpreted as the performance capability of the item being evaluated, occasionally referred to as “engineering tolerance” or “performance specification”, in addition to others. Unless otherwise specified, that numerator is generally the “maximum allowed error”, which is not to be confused with “actual error”. The designation, error ratio was created in an attempt to facilitate the learning process since this is a new term for a newly documented category of measurement uncertainty evaluation. In the strictest sense, it is the ratio between the maximum error allowed by the specification authority and the maximum certified error of the measurement system, neither of which is likely an actual error. This definition is correct in so far that it, in part, represents current industrial practice. Since current practice cannot be expected to significantly change, the creation of this definition was considered necessary as a basis for this new, real-world category of measurement uncertainty evaluation.

E3.10 Expanded Error

This definition is necessary to describe the total error comprised of the sum of the net systematic error and probabilistically determined random error. This sum is the primary basis for the determination of confidence level, which in Metrology, is one of the two fundamental components of measurement uncertainty, the other being confidence interval.

E3.14 Guardband Limit

As currently defined in Metrology, the acceptance limit is being reduced. The specification limit remains what it is, the value of either the requirement or capability. Viewed positively, the bilateral acceptance limit can vary from zero to the full specification limit, i.e., if zero, the measurand estimate can be accepted for measured values as large as the specification limit; if the full specification limit, the measurand estimate can be accepted only for a measured value equal to the certified value. Guardband limit, as defined here, is risk-based and is therefore consistent with the general basis of this document.

E3.15 Historical Measurand Estimate Uncertainty

This definition facilitates the accurate calculation of consumer and producer risks. As such, it avoids confusion, albeit at the addition of a new definition. The confidence level of the estimate of the measurand refers to the percentage of the area under the probability density function model representing the historical measurand estimate values, for a specific confidence interval, i.e., for either a biased or unbiased measurand estimate Gaussian density function, for a confidence interval of ± 2 sigma, the confidence level is 95.45%. For the less accurate option, i.e., Error Ratio, Conservative Option, it is assumed that the maximum allowable error, as represented by the specification limits, has an associated normal product distribution centered between the specification limits. In the absence of a required confidence interval, the product distribution confidence interval is defined to be equivalent to the intended (calibration certified performance specification) value. In the absence of distribution biases, the associated statistics are conservative. In the presence of biases, they can be liberal or conservative.

E3.21 M&TE Performance Specification Tolerance

Maximum allowable error is the chosen terminology in this AIR, and is generally associated with an engineering requirement, whereas, performance specification (tolerance) here represents the item's capability. Both are needed to preserve the distinction between them. Maximum error, as defined by the VIM, can apply to both, therefore blurring the distinction. There is a place for all three in the appropriate contexts. Unless specifically requested, the capability of a measuring device is based on its performance in the evaluation environment and all its particulars.

E3.22 Measurement System Expanded Uncertainty

The GUM definition is not as definitive or useful as the AIR definition and its expected users. The AIR appendices contain the necessary significant qualifying information contained in GUM notes to the definition.

E3.23 Measurement System Total Error

This definition is needed since the Measurement System Total Error is used to calculate error ratio.

APPENDIX A CONFIDENCE LEVEL TRANSFORMATION

Rationale

This Appendix is necessary to ensure the correct calculation of Uncertainty Ratio.

Example

An error ratio (ER) or uncertainty ratio (UR) must be based on the same confidence level (CL) to be valid. If not, any derived statistic expressed as a function of the CL will be in error.

$$UR = \frac{\text{Historical Measurand Estimate Uncertainty}}{\text{Measurement System Expanded Uncertainty}} \equiv \frac{HMEU}{MSEU}$$

Given:

1. Historical measurand estimate uncertainty¹ Confidence interval (CI) is $\pm 0.005''$.
2. Historical measurand estimate uncertainty Confidence level (CL) is 90% (1.645σ).
3. Historical measurand estimate uncertainty Coverage Factor (k) is 1.645.
4. Measurement System Expanded Uncertainty CI is $\pm 0.001''$.
5. Measurement System Expanded Uncertainty CL is 95% (1.960σ).
6. Measurement System k is 1.960.
7. Incorrect UR equals $0.005/0.001 = 5:1$.
8. Desired CL is 95.45%.

Assumption:

1. Both the historical measurand estimate uncertainty and Measurement System Total Uncertainty have Normal distributions.

Transformation:

$$HMEU_{\text{Desired}} = k_{\text{Desired}} \left(\frac{HMEU_{\text{Given}}}{k_{\text{Given}}} \right) = 2.000 \left(\frac{0.005}{1.645} \right) = 0.00608$$

$$MSEU_{\text{Desired}} = k_{\text{Desired}} \left(\frac{MSEU_{\text{Given}}}{k_{\text{Given}}} \right) = 2.000 \left(\frac{0.001}{1.960} \right) = 0.00102$$

Thus, the HMEU and MSEU CL have been transformed to the same 95.45% CL.

Note: The correctly calculated UR is $\frac{0.00608}{0.00102} = 5.96 : 1$

¹ Refer to Appendices E G and I for additional information on the HMEU.

APPENDIX B UNCERTAINTY RATION (UR)

Rationale

This Appendix explains the steps in the calculation of the Uncertainty Ratio which is necessary for the most accurate estimation of Consumer Risk.

Calculation Process

A. Historical Measurand Estimate Uncertainty

1. Obtain this value from a historical measurand estimate uncertainty evaluation in accordance with Appendix H, "Historical Measurand Estimate Uncertainty".
2. Determine the required, requested or historically accepted confidence level (CL).

B. Measurement System Expanded Uncertainty

1. Obtain this value from a measurement uncertainty evaluation in accordance with Appendix C, "Measurement Uncertainty Evaluation – Uncertainty Ratio".

C. Uncertainty Ratio Calculation

1. Verify that the historical measurand estimate uncertainty CL and measurement system expanded uncertainty CL are the same. If different, one or the other must be transformed in accordance with Appendix A, "Confidence Level Transformation".
2. Divide the historical measurand estimate uncertainty by the measurement system expanded uncertainty.

$$UR = \frac{\text{Historical Measurand Estimate Uncertainty}}{\text{Measurement System Expanded Uncertainty}} \equiv \frac{HMEU}{MSEU}$$

APPENDIX C

EXAMPLE MEASUREMENT UNCERTAINTY EVALUATION (UNCERTAINTY RATIO)²

Rationale

This Appendix offers an example to illustrate the application of the law of propagation of uncertainty incorporating many of the principles of other Appendices. The objective is to ease the implementation of those principles when the worst-case tabular values of guardbands of the AS5870 are too restrictive and an individual evaluation is desired.

1. Overview

This option is intended to be the most rigorous approach to measurement uncertainty evaluation in which uncertainties are treated formally and are not combined with systematic errors, hence the title, "Measurement Uncertainty Evaluation". All significant uncertainty sources are random in nature. This approach is generally reserved for the most demanding requirements and for situations where the expanded uncertainty, based on general performance specifications, must be reduced. This approach is consistent with the GUM, reference Appendix N [1]. Note: As a consequence, the consumer and producer risks are as correct as the assumptions and models allow them to be correct. The major steps are as follows:

- A. Document the measurement process..
- B. Identify the uncertainty sources.
- C. Determine the measurement uncertainty process type.
- D. Develop the mathematical model of the measurement process and quantify the values of the measurement system input quantities.
- E. Identify, quantify and correct all known significant sources of measurement system input quantity systematic error.
- F. Identify and quantify all known significant sources of measurement system input quantity random error as standard uncertainties.
- G. Develop the uncertainty equation for the standard uncertainties.
- H. Calculate the combined standard uncertainty of the measurement system input quantity standard uncertainties.
- I. Calculate the coverage factor for the measurement system combined standard uncertainty.
- J. Calculate the expanded uncertainty for the measurement system output quantity.
- K. Evaluate the need to guardband.
- L. Report the error.

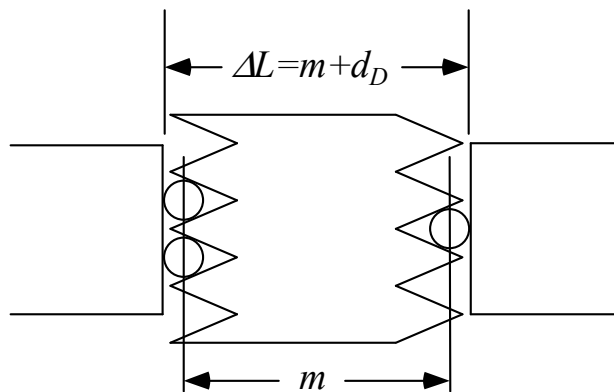
² This uncertainty evaluation is not designed to be a fully detailed and rigorously evaluated example, and should not be considered a fully sufficient model. It is offered only to provide a practical description of the Type 3 measurement uncertainty evaluation process applied to thread pitch diameter measurement.

2. Details

The following is based on the European Co-operation for Accreditation EA-10/10, "EA Guidelines on the Determination of Pitch Diameter of Parallel Thread Gauges by Mechanical Probing, April 1999".

A. Document the Measurement Process.

A 0.3750x16 external thread plug gage pitch diameter is measured over wires. The thread member and wires are placed between the flat anvils of a linear measuring machine (LMM) certified to $\pm 40\mu$ at an overall level of confidence (CL) of 95.45% with a corrected systematic error. The LMM CL is based on a t distribution of a sample size of 10. Each wire has been certified to $\pm 10\mu$ at a CL of 95.45% with no significant systematic errors. The wire CL is based on an assumed normal (Gaussian) distribution. It is desired to certify the pitch diameter to as small an uncertainty as possible at a CL of 95.45%, and not exceed a consumer risk of 0.8% after correction of systematic errors.



B. Identify the Uncertainty Sources

- $u(\Delta L)$ standard uncertainty of measured displacement
- $u(d_D)$ standard uncertainty of calibrated value of probing element diameter
- $u(P)$ standard uncertainty of pitch measurement
- $u(\alpha)$ standard uncertainty of measurement of the flank angle
- $u(A_1)$ standard uncertainty of rake correction equation approximation
- $u(A_2)$ standard uncertainty of elastic compression equation approximation
- $u(d_B)$ standard uncertainty of form deviations
- $u(C_L)$ standard uncertainty of length correction

The distribution of the measurand is assumed to be normal (Gaussian).

C. Determine the Uncertainty Process Type

Since it is desired to certify the pitch diameter to as small an uncertainty as possible, and since there are multiple sources of uncertainty, a Type 1 or Type 2 Conservative Error Ratio process cannot be selected. Since a significant systematic error is present, which can and will be corrected, a Type 3 Uncertainty Ratio process is necessary because it is the most rigorous and systematic errors are corrected.

D. Develop the Mathematical Model of the Measurement Process and Quantify the Values of the Measurement System Input Quantities.

The mathematical model is an analytical expression of the measurement process. The measurand is left of the equal sign and is referred to as the output quantity. All terms right of the equal sign are referred to as input quantities. The expression can be transformed to solve for any measurand, depending upon what output quantity is to be evaluated for error.

For this example, there are eight sources of error, most of which are significant. Since the measured diameter over wires is the sum of the wire diameter and the distance between the wires, a math model for this process can be

$$d_2 = \Delta L - d_D \left[\frac{1}{\sin(\alpha/2)} + 1 \right] + \frac{P}{2} \cot(\alpha/2) - A_1 + A_2 + \delta B$$

where³

ΔL	=	0.38889"
d_D	=	0.03608"
P	=	0.06250"
α	=	60.15°
A_1	=	9μ"
A_2	=	8μ"
δ_B	=	7μ"
C_L	=	not applicable
$u(\Delta L)$	=	17.24μ"
$u(d_D)$	=	6μ"
$u(P)$	=	20μ"
$u(\alpha)$	=	0.002μ"
$u(A_1)$	=	5μ"
$u(A_2)$	=	4μ"
$u(\delta_B)$	=	3μ"
$u(C_L)$	=	2μ"

E. Identify, Quantify and Correct all Known Significant Sources of Input Quantity Systematic Error

If it is believed that the magnitude of the systematic error is different since the time or calibration, or if it has been some time since the item producing each input quantity has been calibrated, new calibration data should be obtained.

In this example, it has been determined that the linear measuring machine has a +12μ" systematic error. Each of the wires does not have a significant systematic error.

F. Identify and Quantify all Known Significant Sources of Input Quantity Random Effect as Standard Uncertainties

Random errors remain after the identification of systematic errors. If there are no systematic errors, the performance specification confidence interval represents the statistical bounds of the random effects, which is equivalent to that input quantity's expanded uncertainty.

³ These values are not the result of actual measurements, or of the actual estimation of their uncertainties. They were chosen only to aid in the presentation of this uncertainty evaluation method. There may be other uncertainties to be accounted for.

If necessary, to convert the expanded uncertainty to a standard uncertainty, it is necessary to divide the expanded uncertainty by a characteristic constant (CC). The first step is to identify or assume the measurand probability distribution.

Table 1 provides some distribution selection guidelines. Tables 2 through 7 contain the characteristic constants for different probability distributions and levels of confidence. A statistical t table can be consulted for sample sizes above 9.

TABLE 1 - GUIDELINE EXAMPLES

Uniform Distribution:	(1) Digital resolution, (2) Data truncation
U Distribution:	(1) Heating or cooling temperature cycling, (2) Sinusoidal variability
Triangular Distribution:	(1) Two uniform distributions of equal range, (2) Peaked distribution with limits, e.g., data with a central tendency with limits resulting from no-go gages.
Normal Distribution:	All other possibilities, assuming an infinite sample size (100% confidence level), approximating symmetry about the mean, without limits.
t Distribution:	All other possibilities, with a known finite sample size (<100% confidence level), approximating symmetry about the mean, without limits.

TABLE 2. ASSORTED DISTRIBUTIONS AND CONFIDENCE LEVELS

<u>Probability Distribution</u>	<u>Confidence Level</u>	<u>CC (ranked)</u>
Normal (4σ)	99.99%	4.000
Normal (3σ)	99.73%	3.000
Normal (2.576σ)	99.00%	2.576
Triangular	100.00%	2.449
Normal (2σ)	95.45%	2.000
Normal (1.960σ)	95.00%	1.960
Uniform	100.00%	1.732
Normal (1.645σ)	90.00%	1.645
U (Arcsin)	100.00%	1.414
Normal (1σ)	68.27%	1.000

TABLE 3 - T DISTRIBUTION AND 90% CONFIDENCE LEVEL

<u>Probability Distribution</u>	<u>Confidence Level</u>	<u>CC (ranked)</u>
t (1.645σ , $n=2$)	90.00%	6.31
t (1.645σ , $n=3$)	90.00%	2.92
t (1.645σ , $n=4$)	90.00%	2.35
t (1.645σ , $n=5$)	90.00%	2.87
t (1.645σ , $n=6$)	90.00%	2.02
t (1.645σ , $n=7$)	90.00%	1.94
t (1.645σ , $n=8$)	90.00%	1.89
t (1.645σ , $n=9$)	90.00%	1.86
t (1.645σ , $n=10$)	90.00%	1.83

TABLE 4 - T DISTRIBUTION AND 95% CONFIDENCE LEVEL

<u>Probability Distribution</u>	<u>Confidence Level</u>	<u>CC (ranked)</u>
$t(1.96\sigma, n=2)$	95.00%	12.71
$t(1.96\sigma, n=3)$	95.00%	4.30
$t(1.96\sigma, n=4)$	95.00%	3.18
$t(1.96\sigma, n=5)$	95.00%	2.78
$t(1.96\sigma, n=6)$	95.00%	2.57
$t(1.96\sigma, n=7)$	95.00%	2.45
$t(1.96\sigma, n=8)$	95.00%	2.36
$t(1.96\sigma, n=9)$	95.00%	2.31
$t(1.96\sigma, n=10)$	95.00%	2.26

TABLE 5 - T DISTRIBUTION AND 95.45% CONFIDENCE LEVEL

<u>Probability Distribution</u>	<u>Confidence Level</u>	<u>CC (ranked)</u>
$t(2\sigma, n=2)$	95.45%	13.97
$t(2\sigma, n=3)$	95.45%	4.53
$t(2\sigma, n=4)$	95.45%	3.31
$t(2\sigma, n=5)$	95.45%	2.87
$t(2\sigma, n=6)$	95.45%	2.65
$t(2\sigma, n=7)$	95.45%	2.52
$t(2\sigma, n=8)$	95.45%	2.43
$t(2\sigma, n=9)$	95.45%	2.37
$t(2\sigma, n=10)$	95.45%	2.32

TABLE 6. T DISTRIBUTION AND 99% CONFIDENCE LEVEL

<u>Probability Distribution</u>	<u>Confidence Level</u>	<u>CC (ranked)</u>
$t(2.576\sigma, n=2)$	99.00%	63.66
$t(2.576\sigma, n=3)$	99.00%	9.92
$t(2.576\sigma, n=4)$	99.00%	5.84
$t(2.576\sigma, n=5)$	99.00%	4.60
$t(2.576\sigma, n=6)$	99.00%	4.03
$t(2.576\sigma, n=7)$	99.00%	3.71
$t(2.576\sigma, n=8)$	99.00%	3.50
$t(2.576\sigma, n=9)$	99.00%	3.36
$t(2.576\sigma, n=10)$	99.00%	3.25

TABLE 7. *T* DISTRIBUTION AND 99.73% CONFIDENCE LEVEL

<u>Probability Distribution</u>	<u>Confidence Level</u>	<u>CC (ranked)</u>
$t(3\sigma, n=2)$	99.73%	235.80
$t(3\sigma, n=3)$	99.73%	19.21
$t(3\sigma, n=4)$	99.73%	9.22
$t(3\sigma, n=5)$	99.73%	6.62
$t(3\sigma, n=6)$	99.73%	5.51
$t(3\sigma, n=7)$	99.73%	4.90
$t(3\sigma, n=8)$	99.73%	4.53
$t(3\sigma, n=9)$	99.73%	4.28
$t(3\sigma, n=10)$	99.73%	4.09

In this example:

The LMM has an uncorrected +12μ" systematic error resulting in a remaining random effects distribution based on a *t* distribution of a sample size of 10 with a 95.45% confidence level based on the distribution only. The LMM standard uncertainty equals its expanded uncertainty divided by its characteristic constant (from Table 5), which is equal to 40μ" divided by 2.32, yielding 17.24μ".

All other uncertainty sources are random in nature. Since their uncertainties are already expressed as standard uncertainties, conversion from expanded uncertainties is unnecessary.

All three wires are assumed to be perfectly correlated. This is assumed to have been accounted for in the sensitivity coefficient term for d_D in the math model. All other quantities are assumed to be uncorrelated.

G. Develop the Uncertainty Equation for the Standard Uncertainties.

A valid uncertainty evaluation must begin with a mathematical model. After the math model is documented, the uncertainty equation for the standard uncertainties can be considered. Different functional relationships will require different treatments in developing the uncertainty equation from the math model. The exact solution to obtaining an analytical expression for the uncertainty is beyond the scope of this document, however, it can be coarsely approximated by applying the law of propagation of uncertainty to the uncertainty equation. In this example, the math model is

$$d_2 = \Delta L - d_D \left[\frac{1}{\sin(\alpha/2)} + 1 \right] + \frac{P}{2} \cot(\alpha/2) - A_1 + A_2 + \delta B.$$

The following uncertainty equation has been developed in accordance with the law of propagation of uncertainty based on the knowledge that linearly related non-correlated input quantities can be Root-Sum-Squared (RSSd). In addition, there is an uncertainty associated with the estimate of the systematic error $u(C_{LMM})$, which must be added to the model. The pitch diameter uncertainty is expressed as a combined standard variance. The positive square root of the combined standard variance is equal to the combined standard uncertainty.

$$u_c^2(d_2) = c_{\Delta L}^2 u^2(\Delta L) + c_{d_D}^2 u^2(d_D) + c_P^2 u^2(P) + c_{\alpha}^2 u^2(\alpha) \\ + c_{A_1}^2 u^2(A_1) + c_{A_2}^2 u^2(A_2) + c_{\delta B}^2 u^2(\delta B) + c_{C_L}^2 u^2(C_L)$$

where

$$c_{\Delta L}^2 = \left(\frac{\partial d_2}{\partial \Delta L} \right)^2, c_{d_D}^2 = \left(\frac{\partial d_2}{\partial d_D} \right)^2, \dots \text{ are the input quantity sensitivity coefficients.}$$

H. Calculate the Combined Standard Uncertainty of the Input Quantity Standard Uncertainties

This is a straightforward process simply requiring replacing variables by their actual values. For the first example above, given the following values:

$$u_c^2(d_2) = (1)^2(17.24)^2 + (3)^2(5)^2 + (0.864)^2(20)^2 + (15)^2(\approx 0)^2 \\ + (-1)^2(5)^2 + (1)^2(4)^2 + (1)^2(3)^2 + (1)^2(2)^2$$

$$u_c^2(d_2) = 874.8$$

$$u_c(d_2) = 29.6 \mu''$$

Assumption:

Both the measurand uncertainty and measurement system input quantities have Normal distributions except the LMM (ΔL), which has a t distribution.

I. Calculate the Coverage Factor for the Combined Standard Uncertainty

The coverage factor is taken as 2 for a confidence level of 95.45% because of the two relatively large Normally distributed standard uncertainties besides the LMM. The Welch-Satterthwaite equation could be used to verify this.

J. Calculate the Expanded Uncertainty of the Measurement System Output Quantity

The expanded uncertainty is equal to the coverage factor times the combined standard uncertainty, e.g.,

$$U = ku_c = 2(29.6\mu'') = 59.2\mu''$$

K. Evaluate the Need to Guardband

Uncertainty Ratio

The UR can now be calculated.

$$UR = \frac{\text{Historical Measurand Estimate Uncertainty}}{\text{Measurement System Expanded Uncertainty}} \equiv \frac{HMEU}{MSEU}$$

$$\text{The UR is } \frac{120}{59.2} = 2.03 : 1$$

Historical Quality Level

The historical quality level (HQL) is calculated to be 90%.

Guardbanding

Based on a desired consumer risk of 0.80%, and HQL of 90%, and an UR of 2.03:1, guardbanding is required since the UR is less than 9.20:1. Linearly interpolating⁴ between the guardband limits (GBL) for uncertainty ratios (URs) of 2:1 and 3:1 from the appropriate table, reproduced below, the required GBL is 0.748.

HQL	UR	GBL	CR(%)	GBL	CR(%)
90%	9.20:1	1.0000	0.8006	1.0000	0.8006
(1.6449σ)	4:1	1.0000	1.6000	0.9210	0.8006
	3:1	1.0000	1.9709	0.8656	0.8006
	2:1	1.0000	2.5462	0.7451	0.8006
	1:1	1.0000	3.5019	0.3726	0.8006

Note: Guardbanding is not required for any consumer risk of 0.8006% or less, i.e., for uncertainty ratios of 9.20:1 or greater.

L. Report the Uncertainty

The thread plug gage can be certified to $\pm 120\mu$ " at a 95.45% CL provided the measured pitch diameter is within $\pm 89\mu$ " ($\pm 0.748 \times 120\mu$ ") of the nominal pitch diameter. Additional helpful information might include:

- Conditions of use that can significantly affect the value of total error.

⁴ Linear interpolation is slightly conservative compared to the actual curve of the GBL versus UR since the curve is concave downward.