



# AEROSPACE STANDARD

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**(R) Data Dictionary for Quantities Used in Cyber-physical Systems**

## RATIONALE

This revision provides further definitions and concepts to support multiple engineering applications.

## FOREWORD

This data dictionary provides definitions for quantities commonly used in the command and control of cyber-physical systems. It defines mathematical and logical terms, quantities, measurement units, reference systems, geometrics, measurands, and measurements. It also defines common quantity modalities. The dictionary is structured to be convenient to data modelers. It is also extendable so that users can create their own quantity domains.

In this data dictionary, a quantity is defined as a property of a phenomenon, substance, or body whose value has numerical magnitude. In contrast, a property whose value does not have numerical magnitude is a nominal property. For example, an identity number is an instance of a nominal property and the length of 0.304 meters is an instance of a quantity.

A measurand specification describes the phenomenon, body or substance and the quantity to be measured. For example, the measurand of aircraft airspeed based on the velocity quantity. A measurement is the set of operations having the object of determining the value of the quantity.

A quantity modal qualifies the semantics of a quantity value. Four modalities are defined that concern the necessity or possibility of a value, the obligation or permissibility of a value, the uncertainty of a value, and derived quantities based on an a-priori waveform model (for example, the values associated with a quantity phasor).

At the highest level, a quantity may be a cardinal quantity or an ordinal quantity depending on whether the numerical part of the value is a cardinal number or an ordinal number respectively. Cardinal quantities may be used in algebraic operations. Ordinal quantities may be used in empirical relationships only. The previous example of length is a cardinal quantity. An example of an ordinal quantity is the octane rating of a liquid fuel for spark-ignition internal combustion engines. The value of a cardinal quantity can be presented in many different mathematical structures including the categories of scalars, true vectors, affine vectors, and true tensors. Except for scalars, these categories are defined by coordinate reference systems (or the transformations between them). Other mathematical categories such as topological spaces can be defined by users if necessary. A coordinate reference system fixes the axes of a coordinate system to the axes of a coordinate datum.

For guidance, Section 3 provides a comprehensive overview of structure and mathematics of this data dictionary. Section 11 provides advice on data modeling considerations.

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## 1. SCOPE

This data dictionary provides definitions for quantities, measurement units, reference systems, measurands, measurements, and quantity modalities commonly used in the command and control of cyber-physical systems. A cyber-physical system is an engineered system that is built from, and depends upon, the seamless integration of computational algorithms and physical components. Cyber-physical systems are often interconnected via data links and networks. The term encompasses intelligent vehicles and devices that operate in any environment, including robotic and autonomous systems.

### 1.1 Purpose

This data dictionary may be used in the development and cross-comparison of data models and in general system and software engineering. The data dictionary can also be extended by users through the creation of quantity domains.

Considerable interoperability can be achieved between different data models when these models reference the same definition URIs provided in this dictionary or its extensions. This includes data models based on different modeling languages and annotated with different spoken languages.

### 1.2 Limitations

This data dictionary is concerned with state data exchanged during the operational activities of cyber-physical systems. It is not concerned with system design specifications or parameters. This data dictionary is not concerned with the structure of data types or classes in a data model.

### 1.3 Viewpoint

In a data model, a class or data type may own properties. A quantity is a property whose value has a numerical magnitude. For example, the class *Aircraft* may own the property of *airspeed* whose basic quantity is velocity. In this example, the UML notation for the measurand specification is *Aircraft:: airspeed: Velocity*.

Specific measurands may be defined in extensions to this standard. These extensions are called quantity domains. This core standard defines basic quantities together with their mathematical structures. For example, the basic quantity of velocity may be expressed in different vector spaces and measurement units. This core standard also defines the general definitions and structure of a quantity domain.

In a cyber-physical system, a quantity value may have multiple modalities. A categorical value is the observable state of the measurand. Other modalities can express possible or necessary values, permissible or obligatory values, the uncertainty of a value, and so on. These modalities are defined in this core standard and would be applied in specific data models that reference this data dictionary.

## 2. REFERENCES

### 2.1 Applicable Documents

The following publications form a part of this document to the extent specified herein. The latest issue of SAE publications shall apply. The applicable issue of other publications shall be the issue in effect on the date of the purchase order. In the event of conflict between the text of this document and references cited herein, the text of this document takes precedence. Nothing in this document, however, supersedes applicable laws and regulations unless a specific exemption has been obtained.

#### 2.1.1 SAE Publications

Available from SAE International, 400 Commonwealth Drive, Warrendale, PA 15096-0001, Tel: 877-606-7323 (inside USA and Canada) or +1 724-776-4970 (outside USA), [www.sae.org](http://www.sae.org).

TSB 003      Rules for SAE Use of SI (Metric) Units

### 2.1.2 ISO Publications

Copies of these documents are available online at <https://webstore.ansi.org/>.

ISO 8601                      Date and time format

ISO/IEC/IEEE 9945      Information technology - Portable Operating System Interface (POSIX®) Base Specifications

ISO 80000                  Quantities and units

Part 1: General

Part 2: Mathematical signs and symbols to be used in the natural sciences and technology

Part 3: Space and time

Part 4: Mechanics

Part 5: Thermodynamics

Part 7: Light

Part 8: Acoustics

Part 9: Physical chemistry and molecular physics

Part 10: Atomic and nuclear physics

Part 11: Characteristic Numbers

Part 12: Condensed Matter Physics

### 2.1.3 IEC Publications

Available from IEC Central Office, 3, rue de Varembe, P.O. Box 131, CH-1211 Geneva 20, Switzerland,  
Tel: +41 22 919 02 11, [www.iec.ch](http://www.iec.ch).

IEC 80000                  Quantities and units

Part 6: Electromagnetism

Part 13: Information Science and Technology

### 2.1.4 NIST Publications

Available from NIST, 100 Bureau Drive, Stop 1070, Gaithersburg, MD 20899-1070, Tel: 301-975-6478, [www.nist.gov](http://www.nist.gov).

NIST Handbook 44: Specifications, Tolerances, and Other Technical Requirements for Weighing and Measuring Devices

### 2.1.5 Other Publications

JCGM 100                  Evaluation of measurement data - Guide to the expression of uncertainty in measurement

## 2.2 Related Publications

The following publications are provided for information purposes only and are not a required part of this SAE Technical Report.

### 2.2.1 Other Publications

DoD World Geodetic System 1984, NIMA TR8350.2, Third Edition Amendment 1

## 2.3 Definitions

### 2.3.1 Definitions Concerning Numbers

#### 2.3.1.1 CARDINAL NUMBER

A cardinal number is a countable numeral that expresses an amount of entities or an amount of measurement units.

NOTE: The numerical value of a cardinal quantity is a cardinal number.

#### 2.3.1.2 ORDINAL NUMBER

An ordinal number represents the numerical position of an entity in an ordered series or the numerical position of a measurement on an ordinal measurement scale.

NOTE: The numerical value of an ordinal quantity is an ordinal number.

#### 2.3.1.3 NOMINAL NUMBER

A nominal number is a numeral that represents the identity of an object or represents a measurement value without magnitude. Nominal numbers have no mathematical significance.

NOTE: A nominal number may be the value of a nominal property.

### 2.3.2 Definitions Concerning Quantities

#### 2.3.2.1 QUANTITY

A quantity is the property of a phenomenon, body, or substance where the property has a value with a numerical magnitude. Two types of quantity are defined: cardinal quantity and ordinal quantity.

#### 2.3.2.2 CARDINAL QUANTITY

A cardinal quantity is a quantity that has magnitude that can be expressed by a numeral multiplied with a measurement unit. A cardinal quantity is a scalar. Cardinal quantities may be used in algebraic operations.

NOTE: All quantities currently in the International System of Quantities (ISQ) (refer to ISO/IEC 80000) are cardinal quantities.

#### 2.3.2.3 RELATIVE QUANTITY

A relative quantity is a cardinal quantity whose value is expressed as a ratio with an empirical value of the same cardinal quantity. The value of a relative quantity has a dimension of one.

NOTE: For example, the volume of liquid fuel as a fraction of the volume capacity of a liquid fuel tank.

#### 2.3.2.4 ORDINAL QUANTITY

An ordinal quantity is a quantity for which a total ordering relation can be established according to magnitude with other quantities of the same kind, but for which no algebraic operations among those quantities exist. Ordinal quantities can enter into empirical relations only and have neither measurement units nor quantity dimensions. Ordinal quantities are arranged according to quantity-value scales and are defined by conventional measurement procedures.

#### 2.3.2.5 NOMINAL PROPERTY

A nominal property is a property that has no numerical magnitude. A nominal property has a value, which can be expressed in nominal numbers, words, by alphanumeric codes, or by other means.

NOTE: Nominal properties are beyond the scope of this data dictionary.

#### 2.3.2.6 QUANTITY KIND

A cardinal quantity may be generalized into a quantity kind. All quantities of the same quantity kind have the same quantity dimension. However, not all quantities with the same quantity dimension are of the same quantity kind.

NOTE: As an example, the quantity of length is a quantity kind to which the quantities length of path and position vector belong. Each of these quantities may in turn be a quantity kind for a group of more specialized quantities.

#### 2.3.2.7 SYSTEM OF QUANTITIES

A system of quantities is a set of cardinal quantities together with a set of non-contradictory equations relating those quantities.

#### 2.3.2.8 INTERNATIONAL SYSTEM OF QUANTITIES

The International System of Quantities (ISQ) is the system of cardinal quantities based on the seven base quantities: length, mass, time, electric current, thermodynamic temperature, amount of substance, and luminous intensity. This system of quantities is published in ISO/IEC 80000.

#### 2.3.2.9 BASE QUANTITY

A base quantity is a cardinal quantity in a conventionally chosen subset of a given system of quantities, where no quantity in the subset can be expressed in terms of the other quantities within that subset.

NOTE: Number of entities can be regarded as an additional base quantity in any system of quantities.

#### 2.3.2.10 DERIVED QUANTITY

A derived quantity in a system of quantities is defined in terms of the base quantities of that system.

#### 2.3.2.11 COMPOSITE QUANTITY

In this data dictionary, a composite quantity is quantity that is not in a system of quantities but is composed of quantities that are each within a system of quantities.

#### 2.3.2.12 QUANTITY DIMENSION

A quantity dimension is an expression of the dependence of a cardinal quantity on the base quantities of a given system of quantities as the product of powers of factors corresponding to the base quantities, omitting any numerical factor.

NOTE: In the International System of Quantities, the seven base quantities, plus number of entities, have a single dimension as shown in Table 1. For discussion on the dimensions of derived quantities, see 3.3.

**Table 1 - Base quantities in the ISQ**

| BASE QUANTITY             | SYMBOL FOR DIMENSION |
|---------------------------|----------------------|
| Length                    | L                    |
| Mass                      | M                    |
| Time                      | T                    |
| Electric current          | I                    |
| Thermodynamic temperature | $\Theta$             |
| Amount of substance       | N                    |
| Luminous intensity        | J                    |
| Number of Entities        | 1                    |

The dimension of any derived quantity Q is denoted:

$$\dim Q = L^{\alpha} M^{\beta} T^{\gamma} I^{\delta} \Theta^{\epsilon} N^{\zeta} J^{\eta}$$

### 2.3.2.13 QUANTITY OF DIMENSION ONE

A quantity of dimension one is a quantity for which all the exponents of the factors corresponding to the base quantities in its quantity dimension are zero.

NOTE: Number of entities is a quantity of dimension one. Ratios of quantities of the same quantity kind are quantities of dimension one.

### 2.3.2.14 QUANTITY EQUATION

A quantity equation is the mathematical relation between quantities in a system of quantities, independent of measurement units.

## 2.3.3 Definitions Concerning Measurement Units

### 2.3.3.1 MEASUREMENT UNIT

A measurement unit is a real scalar quantity with which any other quantity of the same quantity kind can be compared to express the ratio of the second quantity to the first one as a number. Measurement units are defined and adopted by convention.

NOTE: A measurement unit for a quantity of dimension one is a number. In some cases, such measurement units are given special names, e.g., radian and decibel, or are expressed as quotients, such as milligrams per gram.

### 2.3.3.2 SYSTEM OF UNITS

A system of units is a set of base units and derived units together with the multiples and submultiples that is defined in accordance with given rules for a given system of quantities.

### 2.3.3.3 COHERENT SYSTEM OF UNITS

A coherent system of units is a system of units that is based on a given system of quantities and for which the measurement unit for each derived quantity is a coherent derived unit.

### 2.3.3.4 INTERNATIONAL SYSTEM OF UNITS

The International System of Units (SI) is the system of units based on the International System of Quantities including their unit names and symbols and a series of unit prefixes and their names and symbols together with rules for their use adopted by the General Conference on Weights and Measures (CGPM).

### 2.3.3.5 U.S. CUSTOMARY MEASUREMENT SYSTEM

The U.S. Customary Measurement System is the system of units based on the yard of length and the pound of mass that is commonly used in the United States and is defined by the National Institute of Standards and Technology (NIST) Handbook 44.

NOTE: Units having the same common names in other countries may differ in magnitude. For example, the U.S. liquid gallon has a different magnitude to the British imperial gallon.

### 2.3.3.6 BASE UNIT

A base unit is a measurement unit that is adopted by convention for a base quantity in a system of quantities.

NOTE: For number of entities, the number one (symbol 1) can be regarded as a base unit in any system of units.

### 2.3.3.7 DERIVED UNIT

A derived unit is a measurement unit for a derived quantity.

### 2.3.3.8 COHERENT DERIVED UNIT

A coherent derived unit is a derived unit that for a given system of quantities and for a chosen set of base units is the product of powers of base units with no other proportionality factor than one.

### 2.3.3.9 OFF-SYSTEM MEASUREMENT UNIT

An off-system measurement unit is a measurement unit that does not belong to a given system of units.

### 2.3.3.10 UNIT EQUATION

A unit equation is a mathematical relation between base units, coherent derived units, or other measurement units.

### 2.3.3.11 CONVERSION FACTOR BETWEEN UNITS

A conversion factor between units is the ratio of two measurement units for cardinal quantities of the same kind.

### 2.3.3.12 MEASUREMENT SCALE

A measurement scale is an ordered set of quantity values of a given quantity kind used in ranking according to magnitude quantities of that kind.

### 2.3.3.13 CONVENTIONAL REFERENCE SCALE

A conventional reference scale is a measurement scale defined by formal agreement.

## 2.3.4 Definitions Concerning Scalars, Vectors, and Tensors

### 2.3.4.1 SCALAR

A scalar is a cardinal quantity having only magnitude and not geometric direction.

NOTE: A scalar quantity is represented in this data dictionary by a symbol of the form  $q$  (in any font and case, depending on the convention for that quantity).

#### 2.3.4.2 VECTOR

A vector is a cardinal quantity that has both magnitude and geometric direction.

NOTE 1: A vector is represented in this data dictionary by a symbol of the form  $\vec{q}$  (in any font and case, depending on the convention for that quantity).

NOTE 2: A vector quantity may be described by a coordinate tuple or by a coordinate matrix, which are both ordered lists of scalar quantities (coordinates) that are related to a common coordinate reference system (CRS).

#### 2.3.4.3 UNIT VECTOR

A unit vector is a vector that has a magnitude of 1 and a measurement unit of 1. A unit vector in the direction of “a” is denoted by the symbol  $\vec{e}_a$ .

NOTE: A vector can be represented as a unit vector multiplied by a scalar quantity. For example, displacement  $\Delta\vec{r} = \Delta r\vec{e}_a$ .

#### 2.3.4.4 COMPONENT VECTOR

In a Cartesian coordinate system, a vector may be described by the summation of its component vectors, where each component vector is in the direction of one of the axes.

NOTE: For example, in a Cartesian coordinate system, displacement  $\Delta\vec{r} = \Delta x\vec{e}_x + \Delta y\vec{e}_y + \Delta z\vec{e}_z$ , where  $\Delta x\vec{e}_x, \Delta y\vec{e}_y, \Delta z\vec{e}_z$  are the component vectors of  $\Delta\vec{r}$  and  $(\Delta x, \Delta y, \Delta z)$  is its coordinate tuple.

#### 2.3.4.5 TENSOR

A tensor is a quantity that describes the linear relations between vectors or other tensors.

NOTE 1: A tensor of the second order is represented in this data dictionary by a symbol of the form  $\vec{\vec{q}}$  (in any font and case, depending on the convention for that quantity). This tensor may be described by a matrix.

NOTE 2: A coordinate transformation matrix is an example of a second-order tensor.

#### 2.3.4.6 COORDINATE SYSTEM

A coordinate system (CS) is a set of mathematical rules for specifying how coordinates are assigned to quantities.

#### 2.3.4.7 VECTOR COORDINATE SYSTEM

A vector CS defines a vector space in which a quantity value is a vector.

NOTE: The Cartesian coordinate tuple  $(0, 0, 0)$  of a vector space denotes the zero vector in that space. For example, zero displacement  $\Delta\vec{r} = (0, 0, 0)$ . It does not denote a position of origin.

#### 2.3.4.8 AFFINE COORDINATE SYSTEM

An affine CS defines an affine space in which a quantity value is a vector related to a position of origin.

NOTE: Euclidean space is an example of an affine space. The Cartesian coordinate tuple  $(0, 0, 0)$  of an affine space denotes the position of origin. For example, position vector  $\vec{r} = (0, 0, 0)$ .

#### 2.3.4.9 AXIS

In this data dictionary, the term axis is defined as a reference direction in a coordinate reference system, coordinate system, or coordinate datum.

NOTE: The vector direction of an axis may be represented by a unit vector. For example, the vector direction of the  $x$ -axis may be represented by  $\vec{e}_x$ .

#### 2.3.4.10 COORDINATE REFERENCE SYSTEM

A coordinate reference system (CRS) is a coordinate system that is anchored to a spatial object by a coordinate datum.

NOTE: This data dictionary defines CRS based on two spatial objects: the Earth and engineering bodies for which a point of origin and reference axes are defined.

#### 2.3.4.11 COORDINATE DATUM

A coordinate datum (CD) is a parameter or set of parameters that define the position of origin and orientation of a coordinate system within a CRS.

NOTE: The position of origin is only used in an affine CS. The orientation is used by both a vector CS and affine CS.

#### 2.3.4.12 COORDINATE OPERATION

A coordinate operation is a change of coordinates, based on a one-to-one relationship, from one CRS to another.

#### 2.3.4.13 COORDINATE TRANSFORMATION

A coordinate transformation is a coordinate operation (often expressed as a transformation matrix) in which the two CRS are based on different coordinate datums.

#### 2.3.4.14 COORDINATE CONVERSION

A coordinate conversion is a coordinate operation in which both CRS are based on the same CD.

### 2.3.5 Definitions Concerning Statistical Quantities

#### 2.3.5.1 EXPECTED VALUE

The expected value of a quantity  $q$  that is a continuous random variable with the probability density function  $f(q)$  is defined as:

$$E(q) = \int_{-\infty}^{\infty} qf(q) dq$$

NOTE: The arithmetic mean of an infinite series of quantity values will converge on the expected value.

#### 2.3.5.2 STANDARD DEVIATION

The standard deviation of a quantity  $q$  that is a continuous random variable with an expected value  $E(q)$  is defined as:

$$\sigma_q = \sqrt{E[(q - E[q])^2]}$$

### 2.3.5.3 VARIANCE

The variance of a quantity  $q$  that is a continuous random variable with a standard deviation  $\sigma_q$  is defined as:

$$\text{var}(q) = \sigma_q^2$$

### 2.3.5.4 COVARIANCE

The covariance of quantities  $q_1$  and  $q_2$  is defined as:

$$\text{cov}(q_1, q_2) = E[(q_1 - E[q_1])(q_2 - E[q_2])]$$

NOTE:  $\text{cov}(q_1, q_2)$  can be expressed as  $\sigma_{q_1, q_2}$ .

## 2.3.6 Definitions Concerning Measurements

### 2.3.6.1 MEASUREMENT

The set of operations having the object of determining the value of a quantity.

### 2.3.6.2 MEASURAND

The particular quantity subject to measurement.

NOTE: The measurand comprises the general quantity to be measured (e.g., defined in Section 4) and the specification of the phenomenon, body, or substance that is the subject of the measurement.

### 2.3.6.3 PRINCIPLE OF MEASUREMENT

The scientific basis or context of measurement.

NOTE: For example, the Doppler effect applied to the measurement of velocity.

### 2.3.6.4 METHOD OF MEASUREMENT

The logical sequence of operations, described generically, used in the performance of measurements.

### 2.3.6.5 MEASUREMENT PROCEDURE

The set of operations, described specifically, used in the performance of particular measurements according to a given method.

### 2.3.6.6 INFLUENCE QUANTITY

A quantity that is not the measurand but that affects the result of the measurement.

NOTE: In accordance with JCGM 100, the definition of influence quantity is understood to include values associated with measurement standards, reference materials, and reference data upon which the result of a measurement may depend, as well as phenomena such as short-term measuring instrument fluctuations and quantities such as ambient temperature, barometric pressure, and humidity.

## 2.3.7 Definitions Concerning Quantity Modalities

### 2.3.7.1 QUANTITY MODAL

A quantity modal is a logical or mathematical operator that qualifies the truth-semantics of a quantity value.

### 2.3.7.2 QUANTITY MODALITY

A quantity modality is a system of quantity modals.

NOTE: A modality may be concerned with necessity and possibility, obligation and permission, uncertainty, past and future values, the properties of a set of quantity values, and so on.

### 2.3.7.3 CATEGORICAL QUANTITY

The term used to make explicit that a quantity definition does not have a modal. The value of categorical quantity can be directly observed and measured.

## 2.4 Mathematical Symbols

### 2.4.1 Symbols for Cardinal Quantities

The symbols for cardinal quantities are arranged into tables that correspond with the parts of ISO 80000. The listing is alphabetical within each table. Where possible, the symbols follow ISO/IEC 80000, which sometimes results in a symbol being used for more than one quantity (but not within the same table). The symbols for quantities are always Times Roman italic.

NOTE: Quantities that are vectors have a scalar symbol in bold text followed by a vector symbol (in parentheses). Symbols for vector components and coordinates are not provided unless these symbols have been established by convention.

**Table 2 - Symbols for numbers of entities**

| QUANTITY           | SYMBOL |
|--------------------|--------|
| Number of entities | 1      |
| Power Ratio        | $L_p$  |
| Control Parameter  | $p$    |

NOTE: In accordance with ISO 80000-1, number of entities can be added to any system of quantities.

**Table 3 - Symbols for basic quantities of space and time**

| QUANTITY                                     | SYMBOL                     |
|----------------------------------------------|----------------------------|
| acceleration                                 | $\mathbf{a}$ ( $\vec{a}$ ) |
| angle                                        | $\theta$                   |
| - angle of deflection in spherical CS        | $\vartheta$                |
| - angle of elevation in spherical (RAE) CS   | $\varphi$                  |
| - angle of latitude in ellipsoidal CS        | $\varphi$                  |
| - angle of rotation in any coordinate system | $\lambda$                  |
| - Euler angle, first rotation                | $\Psi$                     |
| - Euler angle, second rotation               | $\Theta$                   |
| - Euler angle, third rotation                | $\Phi$                     |
| angular acceleration of plane angle $\theta$ | $\alpha$                   |
| angular velocity of plane angle $\theta$     | $\omega$                   |
| area                                         | $A$                        |
| displacement                                 | $\Delta \vec{r}$           |
| distance                                     | $r$                        |
| duration                                     | $t$                        |
| ellipsoidal height (in ellipsoidal CS)       | $h$                        |
| ellipsoidal height rate                      | $v(h)$                     |
| length                                       | $l$                        |
| length of path                               | $s$                        |
| position                                     | $\vec{r}$                  |
| position coordinates (in Cartesian CS)       | $x, y, z$                  |
| radius                                       | $\rho$                     |
| period duration                              | $T$                        |
| rotation                                     | $N$                        |
| rotational frequency                         | $\eta$                     |
| solid angle                                  | $\Omega$                   |
| velocity                                     | $\mathbf{v}$ ( $\vec{v}$ ) |
| volume                                       | $V$                        |

**Table 4 - Symbols for basic quantities of mechanics**

| QUANTITY            | SYMBOL                     |
|---------------------|----------------------------|
| density             | $\rho$                     |
| dynamic viscosity   | $\eta$                     |
| force               | $\mathbf{F}$ ( $\vec{F}$ ) |
| impulse             | $\mathbf{I}$ ( $\vec{I}$ ) |
| kinematic viscosity | $\nu$                      |
| kinetic energy      | $E_k$                      |
| mass                | $m$                        |
| mass change rate    | $q_m$                      |
| mechanical energy   | $E$                        |
| mechanical power    | $P$                        |
| mechanical work     | $W$                        |
| moment of force     | $\mathbf{M}$ ( $\vec{M}$ ) |
| momentum            | $\mathbf{p}$ ( $\vec{p}$ ) |
| potential energy    | $E_p$                      |
| torque              | $M_Q$                      |
| volume change rate  | $q_V$                      |

**Table 5 - Symbols for basic quantities of thermodynamics**

| QUANTITY                  | SYMBOL |
|---------------------------|--------|
| Celsius temperature       | $t$    |
| relative humidity         | $\phi$ |
| thermodynamic temperature | $T$    |

**Table 6 - Symbols for basic quantities of electromagnetism**

| QUANTITY                       | SYMBOL                     |
|--------------------------------|----------------------------|
| capacitance                    | $C$                        |
| electric charge                | $Q$                        |
| electric charge density        | $\rho_V$                   |
| electric current               | $I$                        |
| electric current density       | $\mathbf{J}$               |
| electric field strength        | $\mathbf{E}$ ( $\vec{E}$ ) |
| electric potential             | $V$                        |
| electric potential difference  | $V_{ab}$                   |
| electrical conductance         | $G$                        |
| electrical conductivity        | $\sigma$                   |
| electrical resistance          | $R$                        |
| electrical resistivity         | $\rho$                     |
| inductance                     | $L$                        |
| instantaneous electrical power | $p$                        |
| linked inductance              | $\Psi$                     |
| magnetic flux                  | $\Phi$                     |
| magnetic flux density          | $\mathbf{B}$ ( $\vec{B}$ ) |
| magnetic vector potential      | $\mathbf{A}$ ( $\vec{A}$ ) |
| source voltage                 | $U_s$                      |
| voltage                        | $U$                        |

**Table 7 - Symbols for basic quantities of light**

| QUANTITY                     | SYMBOL                  |
|------------------------------|-------------------------|
| absorptance                  | $\alpha$                |
| emissivity                   | $\varepsilon$           |
| illuminance                  | $E_V$                   |
| irradiance                   | $E_e$                   |
| luminance                    | $L_V$                   |
| luminous efficacy            | $K$                     |
| luminous efficiency          | $V$                     |
| luminous flux                | $\Phi_V$                |
| luminous intensity           | $I_V$                   |
| radiant energy               | $Q_e$                   |
| radiant power                | $\Phi$                  |
| radiant exitance             | $M_e$                   |
| radiant intensity            | $I_e$                   |
| spectral luminous efficacy   | $K(\lambda)$            |
| spectral luminous efficiency | $V(\lambda)$            |
| spectral radiant power       | $\Phi_\lambda(\lambda)$ |
| spectral radiant intensity   | $I_\lambda(\lambda)$    |

**Table 8 - Symbols for basic quantities of physical chemistry and molecular physics**

| QUANTITY                                 | SYMBOL   |
|------------------------------------------|----------|
| amount of substance X concentration      | $c_X$    |
| amount of substance X                    | $n$      |
| molecular concentration (of substance X) | $C_X$    |
| mass of substance (X)                    | $m_X$    |
| mass concentration (of substance X)      | $\rho_X$ |
| mass fraction (of substance X)           | $w_X$    |
| number of particles (of substance X)     | $N_X$    |

**Table 9 - Symbols for basic quantities of atomic and nuclear physics**

| QUANTITY                | SYMBOL              |
|-------------------------|---------------------|
| absorbed dose           | $D$                 |
| absorbed-dose rate      | $\dot{D}$           |
| dose equivalent         | $H$                 |
| energy of particle      | $E$                 |
| linear energy transfer  | $L_\infty$          |
| mean energy imparted    | $\bar{\varepsilon}$ |
| quality factor          | $Q$                 |
| radiant energy          | $R$                 |
| rest energy of particle | $E_0$               |
| rest mass of particle   | $m_0$               |

**Table 10 - Symbols for basic quantities of information science and technology**

| QUANTITY          | SYMBOL   |
|-------------------|----------|
| binary digit rate | $v$      |
| Clock frequency   | $f_{cl}$ |
| storage capacity  | $M$      |

## 2.4.2 Symbols for Coordinate Systems and Unit Vectors

**Table 11 - Symbols for coordinate system unit vectors**

| QUANTITY                                                    | SYMBOL    |
|-------------------------------------------------------------|-----------|
| eccentricity of ellipse or ellipsoid                        | $e_e$     |
| flattening factor of ellipse or ellipsoid                   | $f_e$     |
| length of semi-major axis of ellipse or ellipsoid           | $a_e$     |
| length of semi-minor axis of ellipse or ellipsoid           | $b_e$     |
| radius of curvature of the local meridian in ellipsoidal CS | $\rho$    |
| radius of curvature of the local parallel in ellipsoidal CS | $\rho$    |
| unit vector                                                 | $\vec{e}$ |

## 2.4.3 Symbols for Quantity Dimensions

**Table 12 - Symbols for quantity dimensions**

| QUANTITY DIMENSION        | SYMBOL   |
|---------------------------|----------|
| length                    | L        |
| mass                      | M        |
| time                      | T        |
| electric current          | I        |
| thermodynamic temperature | $\Theta$ |
| amount of substance       | N        |
| luminous intensity        | J        |
| number of entities        | 1        |

## 2.4.4 Symbols for Measurement Units

The symbols for measurement units are arranged alphabetically into SI units, the units in the U.S. Customary Measurement System, and off-system units. Whereas quantity symbols are Times Roman italic, unit symbols are always in normal text.

NOTE 1: The symbol for a unit may be different from the abbreviation. For example, the symbol for the mile per hour measurement unit is mi/h, while the common abbreviation is mph.

NOTE 2: The measurement unit of one (symbol “1”) is included with the SI measurement units.

**Table 13 - Symbols for SI measurement units**

| MEASUREMENT UNIT         | SYMBOL             |
|--------------------------|--------------------|
| ampere                   | A                  |
| ampere per square meter  | A/m <sup>2</sup>   |
| bar                      | Bar                |
| bit                      | bit                |
| bit per second           | bit/s              |
| byte                     | B                  |
| byte per second          | B/s                |
| candela                  | cd                 |
| candela per square meter | cd m <sup>-2</sup> |
| coulomb                  | C                  |
| coulomb per cubic meter  | C/m <sup>3</sup>   |
| cubic meter              | m <sup>3</sup>     |
| cubic meter per second   | m <sup>3</sup> /s  |
| day                      | d                  |
| decibel                  | dB                 |
| degree of arc            | °                  |
| degree of arc per second | °/s                |
| degree Celsius           | °C                 |
| farad                    | F                  |
| gram                     | g                  |
| gram per liter           | g/l                |
| gray                     | Gy                 |
| gray per second          | Gy/s               |

|                                  |                                     |
|----------------------------------|-------------------------------------|
| henry                            | H                                   |
| hertz                            | Hz                                  |
| hour                             | h                                   |
| joule                            | J                                   |
| kelvin                           | K                                   |
| kilogram                         | kg                                  |
| kilogram meter per second        | kg m/s                              |
| kilogram per cubic meter         | kg/m <sup>3</sup>                   |
| kilogram per second              | kg/s                                |
| kilometer                        | km                                  |
| kilometer per hour               | km/h                                |
| kilopascal                       | kPa                                 |
| liter                            | l                                   |
| lumen                            | lm                                  |
| lumen per watt                   | lm W <sup>-1</sup>                  |
| lux                              | lx                                  |
| meter of length                  | m                                   |
| meter per second                 | m/s                                 |
| meter per second squared         | m/s <sup>2</sup>                    |
| meter squared per second         | m <sup>2</sup> /s                   |
| meter squared per second squared | m <sup>2</sup> /s <sup>2</sup>      |
| meter to the power minus three   | m <sup>-3</sup>                     |
| minute of time                   | min                                 |
| mole                             | mol                                 |
| neper                            | Np                                  |
| newton                           | N                                   |
| newton meter                     | N m                                 |
| ohm                              | Ω                                   |
| ohm meter                        | Ω m                                 |
| one                              | 1                                   |
| parts per thousand               | ppt                                 |
| pascal                           | Pa                                  |
| pascal second                    | Pa s                                |
| percent                          | pc                                  |
| rad                              | rad                                 |
| radian                           | rad                                 |
| radian per second                | rad/s                               |
| radian per second squared        | rad/s <sup>2</sup>                  |
| rem                              | rem                                 |
| second                           | s                                   |
| second to the power minus one    | s <sup>-1</sup>                     |
| siemens                          | S                                   |
| siemens per meter                | S/m                                 |
| sievert                          | Sv                                  |
| square meter                     | m <sup>2</sup>                      |
| steradian                        | sr                                  |
| tesla                            | T                                   |
| volt                             | V                                   |
| volt per meter                   | V/m                                 |
| watt                             | W                                   |
| watt per hertz                   | W Hz <sup>-1</sup>                  |
| watt per steradian               | W sr <sup>-1</sup>                  |
| watt per steradian per hertz     | W sr <sup>-1</sup> Hz <sup>-1</sup> |
| weber                            | Wb                                  |
| weber per meter                  | Wb/m                                |

**Table 14 - Symbols for measurement units in the U.S. customary measurement system**

| MEASUREMENT UNIT               | SYMBOL               |
|--------------------------------|----------------------|
| cubic foot                     | ft <sup>3</sup>      |
| degree Fahrenheit              | °F                   |
| foot per second                | ft/s                 |
| foot per second squared        | ft/s <sup>2</sup>    |
| foot pounds per second squared | ft lb/s <sup>2</sup> |
| inch                           | in                   |
| inch per second                | in/s                 |
| mile per hour (mph)            | mi/h                 |
| pound force                    | lbf                  |
| pound of force per square inch | psi                  |
| pound of mass                  | lb                   |
| pound of mass per cubic foot   | lb/ft <sup>3</sup>   |
| pound of mass per cubic inch   | lb/in <sup>3</sup>   |
| pound of mass per second       | lb/s                 |
| square foot                    | ft <sup>2</sup>      |
| statute foot                   | ft                   |
| statute mile                   | mi                   |
| yard of length                 | yd                   |

**Table 15 - Symbols for off-system measurement units**

| MEASUREMENT UNIT       | SYMBOL       |
|------------------------|--------------|
| inches mercury (60 °F) | inHg (60 °F) |
| revolutions per minute | r/min        |
| year                   | a            |

#### 2.4.5 Mathematical Symbols and Signs

This data dictionary is conformant to ISO 80000-2:2009 in its use of mathematical symbols and signs. The following table is informational only.

**Table 16 - Mathematical symbols and signs**

| MATHEMATICAL MEANING                     | SYMBOL, EXPRESSION       |
|------------------------------------------|--------------------------|
| $a$ is, by definition, equal to $b$      | $a := b$                 |
| $a$ is exactly equal to $b$              | $a = b$                  |
| $a$ is approximately equal to $b$        | $a \approx b$            |
| base of natural logarithm                | $e$                      |
| expected value of $q$                    | $E(q)$                   |
| standard deviation of $q$                | $\sigma_q$               |
| variance of $q$                          | $var(q)$                 |
| covariance of $q_1, q_2$                 | $cov(q_1, q_2)$          |
| vector $a$                               | $\vec{a}$                |
| unit vector                              | $\vec{e}$                |
| tensor $a$                               | $\vec{\vec{a}}$          |
| cross product of $\vec{a}$ and $\vec{b}$ | $\vec{a} \times \vec{b}$ |
| dot product of $\vec{a}$ and $\vec{b}$   | $\vec{a} \cdot \vec{b}$  |
| unit quaternion                          | $h$                      |

NOTE: In Table 16, the symbols  $a, b$  are arbitrary.

### 3. OVERVIEW OF QUANTITIES

This data dictionary is concerned with the definition of quantities, measurement units, reference systems, measurands, measurements, and quantity modalities.

#### 3.1 Numbers

In this data dictionary, three types of number are used: cardinal, ordinal, and nominal. A cardinal number is a numeral that expresses an amount of entities or an amount of measurement units (e.g., 1, 2, 3). An ordinal number is a numeral that expresses the position of an entity in an ordered series (e.g., 1st, 2nd, 3rd) or the position of a measurement on an ordinal quantity-value scale. A nominal number is a numeral that expresses the identity of an entity (e.g., #1, #2, #3) or the numerical value of a measurement that does not have a magnitude.

#### 3.2 Quantities

A quantity is the property of a phenomenon, body, or substance where the property has a value with magnitude. Two types of quantity are identified: cardinal and ordinal. A cardinal quantity has a magnitude that has algebraic meaning and is always a cardinal number multiplied with a measurement unit. An ordinal quantity has a magnitude without algebraic meaning. An ordinal quantity does not have a measurement unit or quantity dimension but is defined according to an established measurement scale. Ordinal quantities may enter into empirical relationships with other quantities but not algebraic relationships.

NOTE: A nominal property has a value without magnitude. The value of a nominal property may be a nominal number.

#### 3.3 Cardinal Quantities

In this data dictionary, cardinal quantities are traceable to the International System of Quantities (ISQ) as defined in ISO/IEC 80000. The ISQ defines seven base quantities (or dimensions) as shown in Table 17. To the ISQ is added an eighth quantity or dimension: number of entities, which has a dimension of one.

**Table 17 - Base quantities in the ISQ**

| BASE QUANTITY             | QUANTITY SYMBOL | DIMENSION SYMBOL |
|---------------------------|-----------------|------------------|
| length                    | $l$             | L                |
| mass                      | $m$             | M                |
| time                      | $t$             | T                |
| electric current          | $I$             | I                |
| thermodynamic temperature | $T_{90}$        | Θ                |
| amount of substance       | $n$             | N                |
| luminous intensity        | $I_V$           | J                |
| number of entities        | 1               | 1                |

Derived quantities are defined by a quantity equation and a dimension equation. For example, inductance (symbol  $L$ ) has the following quantity equation <sup>1</sup> and dimension equation:

$$L = \Psi / I$$

$$\dim L = L^2MT^{-2}I^{-1}.$$

It is sometimes the case that a general quantity (called a quantity kind) can be refined into a more specialized quantity. For example, length  $l$  can be refined into length of path  $s$  or position vector  $r$ . Within the context of a given coordinate system, position vector  $r$  can in turn be refined into its component vectors. For example, in a Cartesian coordinate system,  $r$  can be refined into its component lengths  $x$ ,  $y$ , and  $z$ .

<sup>1</sup> In this example,  $I$  is an electric current in a thin conducting loop (a base quantity) and  $\Psi$  is the linked flux caused by that electric current (a derived quantity).

In this data dictionary, a basic cardinal quantity is strictly a quantity that is defined completely by the ISQ or by its components within a given coordinate system. Table 18 is an example of a definition of a basic cardinal quantity (inductance).

**Table 18 - Defining elements of inductance quantity (example)**

| DEFINING ELEMENT                             | DESCRIPTION                                                                                                                              |
|----------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------|
| Quantity name                                | Inductance quantity                                                                                                                      |
| Version                                      | 1.1                                                                                                                                      |
| Definition URI                               | Namespace, UUID conformant to RFC4122 standard, and version                                                                              |
| Quantity type                                | Cardinal                                                                                                                                 |
| System of quantities                         | ISQ                                                                                                                                      |
| Authoritative reference                      | IEC 80000-6:2008                                                                                                                         |
| Quantity kind                                | Inductance quantity                                                                                                                      |
| Quantity symbol                              | $L$                                                                                                                                      |
| Quantity dimension                           | $\dim L = L^2MT^{-2}I^{-1}$                                                                                                              |
| Quantity equation                            | $L = \Psi/I$<br>where $I$ is an electric current in a thin conducting loop and $\Psi$ is the linked flux caused by that electric current |
| Field of application                         | Electromagnetism                                                                                                                         |
| Quantity application                         | General                                                                                                                                  |
| Preferred measurement unit                   | Henry unit                                                                                                                               |
| Other measurement units and unit application | None                                                                                                                                     |
| Measurement procedure or scale               | None                                                                                                                                     |
| Reference material                           | None                                                                                                                                     |

In this data dictionary, basic cardinal quantities are initially organized according to their field of application following ISO/IEC 80000:

- Space and time (refer to ISO 80000 Part 3)
- Mechanics (refer to ISO 80000 Part 4)
- Thermodynamics (refer to ISO 80000 Part 5)
- Electromagnetism (refer to IEC 80000 Part 6)
- Light (refer to ISO 80000 Part 7)
- Acoustics (refer to ISO 80000 Part 8)
- Physical chemistry and molecular physics (refer to ISO 80000 Part 9)
- Atomic and nuclear physics (refer to ISO 80000 Part 10)
- Information science and technology (refer to IEC 80000 Part 13)

NOTE: Each of these fields of application exists within a dimensional space

**Table 19 - Dimensional space of fields of application**

| FIELD OF APPLICATION                     | DIMENSION |   |   |   |   |   |   |   |
|------------------------------------------|-----------|---|---|---|---|---|---|---|
|                                          | 1         | T | L | M | I | Θ | J | N |
| Space and time                           | X         | X | X |   |   |   |   |   |
| Mechanics                                | X         | X | X | X |   |   |   |   |
| Thermodynamics                           | X         | X | X | X |   | X |   |   |
| Electromagnetism                         | X         | X | X | X | X |   |   |   |
| Light                                    | X         | X | X | X |   |   | X |   |
| Acoustics                                | X         | X | X | X |   |   |   |   |
| Physical chemistry and molecular physics | X         | X | X | X |   |   |   | X |
| Atomic and nuclear physics               | X         | X | X | X | X |   |   |   |
| Information Science and Technology       | X         | X |   |   |   |   |   |   |

### 3.4 Ordinal Quantities

Ordinal quantities have magnitude but no dimension or measurement unit. Ordinal quantities are measured empirically using a specified (but arbitrary) measurement procedure and scale, which is, therefore, intrinsic to the measurement semantics. An important distinction between an ordinal quantity and cardinal quantity is the type of numerical value. The numerical value of an ordinal quantity is not a scalar (or more generally a tensor) that can be used in algebraic expressions.

Ordinal quantities typically have very specific empirical applications. Examples of ordinal quantities are:

- **Engine fuel octane rating** - Engine fuel octane rating measured with a test engine according to a specified measurement procedure and scale including, for example, the Research Octane Number (RON), Motor Octane Number (MON), or Anti-Knock Index (AKI or  $(R+M)/2$ ).
- **Metal hardness scale** - Metal indentation hardness measured with a specified indenter and load. Hardness scales include the Rockwell hardness test (scales A through H, K, N, and T).
- **Earthquake magnitude scale** - Magnitude of an earthquake based on an agreed empirical formula or correlation table using the maximum excursion of a Wood-Anderson torsion seismograph and its epicentral distance. Scales include the Richter scale, which implicitly incorporates the attenuative properties of the Southern California crust and mantle.

In general, an ordinal quantity is defined by an authoritative industry standard and, if necessary, a selected measurement procedure and scale. At this time, no ordinal quantities are used in this standard. However, Table 20 is a notional example of an ordinal quantity definition (the Rockwell A hardness scale).

**Table 20 - Defining elements of Rockwell A hardness scale quantity (notional example)**

| DEFINING ELEMENT                   | DESCRIPTION                                                                            |
|------------------------------------|----------------------------------------------------------------------------------------|
| Quantity name                      | Rockwell A hardness scale                                                              |
| Version                            | 1.0                                                                                    |
| Definition IRU                     | Namespace, UUID conformant to RFC4122 standard and version                             |
| Quantity type                      | Ordinal                                                                                |
| Authoritative reference            | ISO 6508-1:2015 (Metallic materials - Rockwell hardness test - Part 1: Test method)    |
| Field of application               | Mechanics                                                                              |
| Quantity application               | Indentation hardness test for hard metals                                              |
| Measurement procedure and scale    | ISO 3738-1:1982 (Hard metals - Rockwell hardness test (scale A) - Part 1: Test method) |
| Scale symbol                       | HRA                                                                                    |
| Reference material (if applicable) | None                                                                                   |

### 3.5 Nominal Properties

A nominal property is a property without magnitude. The value of a nominal property may be expressed non-numerically or by a nominal number. Specific nominal properties are not defined in this data dictionary.

### 3.6 Measurement Units

A measurement unit (or simply a unit) is a scalar that can be multiplied with a cardinal number to denote the magnitude of a cardinal quantity. In this data dictionary, units are traceable to the International System of Units (SI). The SI defines a base unit for each of the base quantities in the ISQ as shown in Table 21. In addition, the unit of one can be used to express number of entities, any derived quantity of dimension one, or the ratio between two quantities with the same dimension equation.

**Table 21 - Base units in the SI**

| BASE UNIT | BASE QUANTITY             | UNIT SYMBOL |
|-----------|---------------------------|-------------|
| meter     | length                    | m           |
| kilogram  | mass                      | kg          |
| second    | time                      | s           |
| ampere    | electric current          | A           |
| kelvin    | thermodynamic temperature | K           |
| mole      | amount of substance       | mol         |
| candela   | luminous intensity        | Cd          |
| one       | number of entities        | 1           |

Derived units in the SI are defined by a unit equation. For example, the henry (symbol H) is defined by the unit equation<sup>2</sup>:

$$1 \text{ H} := 1 \text{ Wb/A}$$

In this data dictionary, non-SI units are defined by a conversion factor from the SI unit. For example, the pound of mass (symbol lb) is defined by the conversion factor:

$$1 \text{ lb} := 0.45359237 \text{ kg}$$

### 3.7 Scalars, Vectors, and Tensors

In mathematical terms, a tensor of zero-order is a scalar, a tensor of the first order is a vector, and a tensor of the second order (or higher orders) is a true tensor. Cardinal quantities can, therefore, be scalars, vectors, or tensors.

#### 3.7.1 Scalar Quantity

A scalar quantity is a quantity that has magnitude but no geometric direction. It is mathematically denoted in this data dictionary by a symbol of the form  $q$ . In this data dictionary, all quantities can be scalars, even when their context is implicitly a vector (for example, force).

#### 3.7.2 Vector Quantity

A vector quantity has both magnitude and geometric direction and, therefore, is always associated with a coordinate reference system (CRS). In this data dictionary, a vector is denoted by a symbol of the general form  $\vec{q}$ . The value of the vector within a given coordinate system can be expressed in compact form as either a coordinate tuple or as a coordinate matrix, where in both cases  $q_1, q_2, q_3$  are the coordinates of  $\vec{q}$ .

The coordinate tuple is of the general form:

$$\vec{q} = (q_1, q_2, q_3)$$

<sup>2</sup> In this example, the symbol Wb denotes a weber (a derived unit), and A denotes an ampere (a base unit).

The coordinate matrix is of the general form:

$$\vec{q} = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix}$$

In equation form, any vector can be mathematically represented by a scalar multiplied by the unit vector  $\vec{e}_q$  in the direction  $q$ . Therefore, in a three-dimensional Cartesian coordinate system:

$$\vec{q} = q \vec{e}_q$$

$$\vec{q} = q_x \vec{e}_x + q_y \vec{e}_y + q_z \vec{e}_z$$

where  $q_x \vec{e}_x$ ,  $q_y \vec{e}_y$ , and  $q_z \vec{e}_z$  are the components of  $\vec{q}$ . In this example, the unit vectors  $\vec{e}_x$ ,  $\vec{e}_y$ ,  $\vec{e}_z$  have a magnitude of one and represent the directions of the orthonormal X-axis, Y-axis, and Z-axis, respectively.

### 3.7.3 Affine Quantity

An affine quantity is a quantity related to an arbitrary but standardized reference. Therefore, two affine quantities cannot be added together. However, subtraction produces the difference. Examples of affine quantities are position vector, system time, and Celsius temperature.

An important example of an affine quantity is position in Euclidean space. In this data dictionary, the quantity kind length ( $l$ ) can be refined into displacement ( $\Delta \vec{r}$ ), which is a vector quantity, and position vector ( $\vec{r}$ ), which is an affine quantity.

### 3.7.4 Second Order Tensor Measurements

A tensor is a measurement that describes the linear relationship between vectors, between vectors and tensors, or between tensors. In this data dictionary, a tensor of the second order is denoted by a symbol and coordinate matrix of the general form:

$$\vec{\vec{q}} = \begin{pmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{pmatrix}$$

In a three-dimensional Cartesian coordinate system, the equation form of the tensor is:

$$\vec{\vec{q}} = q_{xx} \vec{e}_x \vec{e}_x + q_{xy} \vec{e}_x \vec{e}_y + \cdots + q_{zz} \vec{e}_z \vec{e}_z$$

$$\vec{\vec{q}} = \begin{pmatrix} q_{xx} & q_{xy} & q_{xz} \\ q_{yx} & q_{yy} & q_{yz} \\ q_{zx} & q_{zy} & q_{zz} \end{pmatrix}$$

An example of a tensor in this data dictionary is a coordinate transformation matrix between one set of Cartesian axes and another. For example:

$$\vec{q}_2 = \vec{\vec{q}}_{21} \vec{q}_1$$

where  $\vec{q}_1$  and  $\vec{q}_2$  are the same vector described in CRS 1 and CRS 2, respectively, and:

$$\vec{\vec{q}}_{21}$$

is the coordinate transformation from CRS 1 to CRS 2. It will be noted that some quantities of interest to this data dictionary are, in fact, the parameters of such a coordinate transformation matrix.

### 3.8 Spatial Coordinate Systems

In this data dictionary, the difference between a vector quantity and an affine quantity is denoted by the quantity kind, which is part of the quantity definition. For example, displacement ( $\Delta\vec{r}$ ) is a vector quantity and position vector ( $\vec{r}$ ) is an affine quantity.

A given coordinate system (CS) may be used to describe both a true vector quantity and a position vector with its differential (which is always a true vector). This allows, for example, both position and motion to be described in the same CS. This is achieved by diagrammatically and mathematically defining the position vector  $\vec{r}$  and its differential  $d\vec{r}$ . The differential is always described using a right-handed system of orthonormal unit vectors. Thus, the core of a coordinate system is the definition of the coordinates plus the definition of the component vectors of the differential, where each component vector has a unit vector and scalar quantity equation.

To illustrate these concepts, the example of a spherical coordinate system for the position vector  $\vec{r}$  is provided. In the case of a vector quantity, the point O denotes the zero vector and the axes define the direction of the basis vectors  $\vec{e}_x$ ,  $\vec{e}_y$ , and  $\vec{e}_z$  (from the right-hand rule). In the case of an affine quantity (such as position), the point O additionally denotes a position of origin. In both cases, the red dot indicates a coordinate.

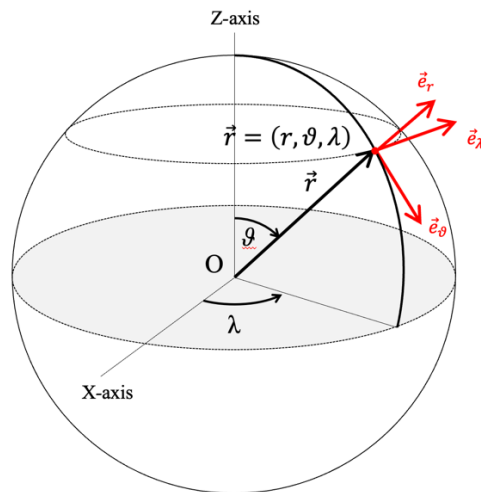


**Figure 1 - Right-handed spherical coordinate system (for vector)**

In this example, the position vector value is represented by the following coordinate tuple and coordinate matrix:

$$\vec{r} = (r, \vartheta, \lambda)$$

$$\vec{r} = \begin{pmatrix} r \\ \vartheta \\ \lambda \end{pmatrix}$$

Where  $\vartheta$  denotes the angle of deflection from the Z-axis and  $\lambda$  denotes the right-handed angle of rotation about the Z-axis from the X-axis. The differential of the position vector,  $d\vec{r}$ , at coordinate  $(r, \vartheta, \lambda)$  is represented by the equation below, where the unit vectors  $\vec{e}_r(\vartheta, \lambda)$ ,  $\vec{e}_\vartheta(\vartheta, \lambda)$ ,  $\vec{e}_\lambda(\lambda)$  form an orthonormal right-handed system as shown in the figure:

$$d\vec{r} = dr \vec{e}_r + r d\vartheta \vec{e}_\vartheta + r \sin \vartheta d\lambda \vec{e}_\lambda$$

Additional vector quantities based on the position differential are defined by quantity equations. For example, in the spherical coordinate system, the velocity,  $\vec{v}$ , of an object at position  $(r, \vartheta, \lambda)$  may be represented by the following coordinate tuple and quantity equations:

$$\vec{v} = (v_r, v_\vartheta, v_\lambda)$$

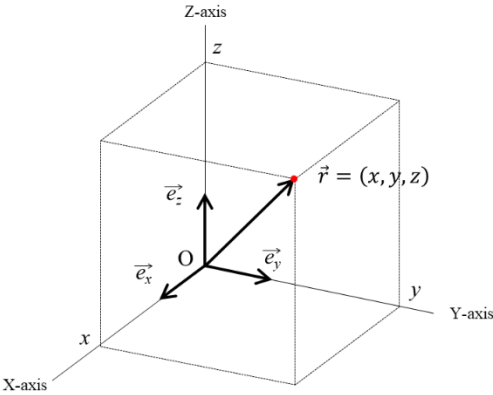
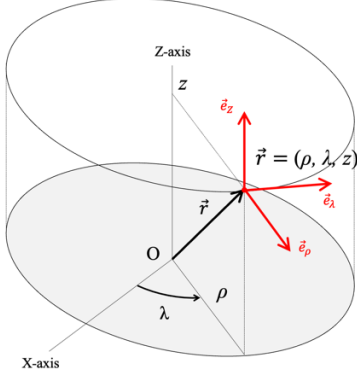
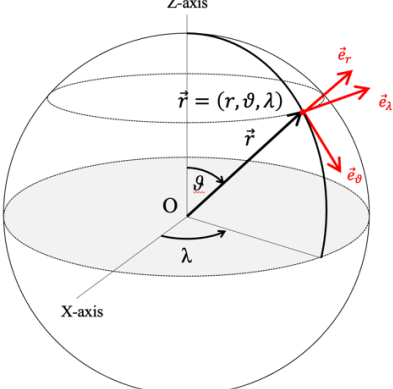
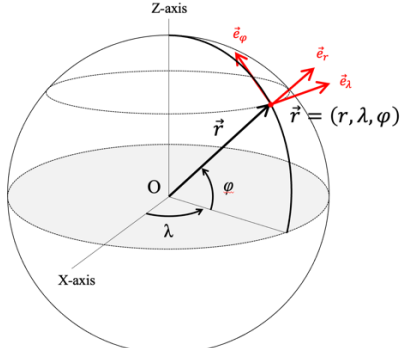
$$\vec{v} = v_r \vec{e}_r + v_\vartheta \vec{e}_\vartheta + v_\lambda \vec{e}_\lambda$$

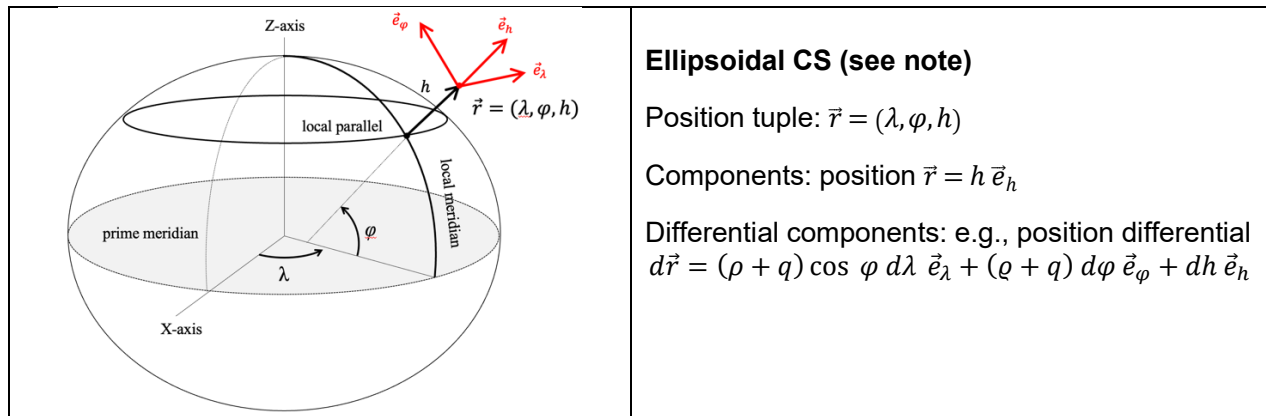
where:

$$v_r = \frac{dr}{dt}, v_\vartheta = r \frac{d\vartheta}{dt}, v_\lambda = r \sin \vartheta \frac{d\lambda}{dt}$$

In this data dictionary, a coordinate system defines quantities and quantity equations. It does not define the measurement unit for each quantity or any other properties that depend on the measurement unit, such as range or resolution. The basic three-dimensional coordinate systems defined in this data dictionary are summarized in Table 22.

**Table 22 - Summary of 3D right-handed coordinate systems**

|                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                   |
|-------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|    | <p><b>Cartesian vector CS</b></p> <p>Quantity vector tuple: <math>\vec{q} = (q_x, q_y, q_z)</math></p> <p>Position tuple: <math>\vec{r} = (x, y, z)</math></p> <p>Components: <math>\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z</math></p> <p>Differential components:<br/> <math>d\vec{r} = dx\vec{e}_x + dy\vec{e}_y + dz\vec{e}_z</math></p>                                                |
|   | <p><b>Cylindrical vector CS</b></p> <p>Quantity vector tuple: <math>\vec{q} = (q_\rho, \lambda, q_z)</math></p> <p>Position tuple: <math>\vec{r} = (\rho, \lambda, z)</math></p> <p>Components: <math>\vec{r} = \rho\vec{e}_\rho + z\vec{e}_z</math></p> <p>Differential components:<br/> <math>d\vec{r} = d\rho \vec{e}_\rho + \rho d\lambda \vec{e}_\lambda + dz \vec{e}_z</math></p>           |
|  | <p><b>Spherical vector CS</b></p> <p>Quantity vector tuple: <math>\vec{q} = (q, \vartheta, \lambda)</math></p> <p>Position tuple: <math>\vec{r} = (r, \vartheta, \lambda)</math></p> <p>Components: <math>\vec{r} = r \vec{e}_r</math></p> <p>Differential components:<br/> <math>d\vec{r} = dr \vec{e}_r + r d\vartheta \vec{e}_\vartheta + r \sin \vartheta d\lambda \vec{e}_\lambda</math></p> |
|  | <p><b>Spherical (RAE) vector CS</b></p> <p>Quantity vector tuple: <math>\vec{q} = (q, \lambda, \varphi)</math></p> <p>Position tuple: <math>\vec{r} = (r, \lambda, \varphi)</math></p> <p>Components: <math>\vec{r} = r \vec{e}_r</math></p> <p>Differential components:<br/> <math>d\vec{r} = dr \vec{e}_r + r \cos \varphi d\lambda \vec{e}_\lambda + r d\varphi \vec{e}_\varphi</math></p>     |



NOTE: The ellipsoidal CS is used for geodetic positioning and has no general application for the quantity  $q$  in practice. The ellipsoidal CS is based on an ellipsoid with a specific magnitude and eccentricity. Its magnitude is defined by the length of its semi-major axis  $a_e$ , and its eccentricity is defined by a second defining parameter, which can be:

- length of its semi-minor axis  $b_e$
- flattening factor  $f_e$ , where  $f_e = 1 - b_e/a_e$
- eccentricity  $e_e$ , where  $e_e^2 = 1 - b_e^2/a_e^2$

The equation for the differential components depends on the radius of curvature,  $\rho$ , of the local parallel and the radius of curvature,  $q$ , of the local meridian, where:

$$\rho = \frac{a_e}{\sqrt{1 - e_e^2 \sin^2 \varphi}}$$

$$q = \frac{a_e(1 - e_e^2)}{(1 - e_e^2 \sin^2 \varphi)^{3/2}}$$

### 3.9 Spatial Coordinate Reference Systems

A coordinate reference system (CRS) is a coordinate system (CS) that is anchored to a coordinate datum (CD). A CD has a point of origin and an orthonormal system of axes that are tied to a real body. The CS is anchored to the CD by equating the two sets of axes. In principle, there are 24 ways in which one CS can be anchored to one CD: the first CS axis can be equated to any one of the three CD axes in either the positive or negative direction; the second CS axis can be equated to any one of the two remaining CD axes in either the positive or negative direction; and the third CS axis is then equated to the remaining CD axis in the direction that complies with the right-hand rule.

In this data dictionary, there are five principal classes of coordinate datum to which coordinate systems can be anchored to create coordinate reference systems. They are:

- Terrestrial reference frame (TRF)
- Earth-fixed local reference frame (LRF)
- Body kinematics LRF
- Celestial reference frame (CRF)
- Engineering body

In addition, there is an abstract CD called a polymorphic CD.

A TRF CD is an Earth-fixed global datum for terrestrial objects. In contrast, an Earth-fixed LRF is a local datum based on some arbitrarily chosen Earth-fixed point. A body kinematics LRF is the same as an Earth-fixed LRF except that its Earth position follows the position of the object of interest. However, kinematically, the LRF is considered stationary. An engineering body CD shares the same position and kinematics as the object of interest. A CRF CD is used for non-terrestrial objects such as those in Earth orbit or in deep space.

A polymorphic CD is an abstract CD whose anchor definition is deferred.

### 3.9.1 Terrestrial Reference Frame CRS

The principal application of a TRF CRS is global Earth-fixed positioning. The primary example is a geodetic CRS. To avoid over-generalizing, the following discussion will therefore concentrate on position and the position differential.

This type of CRS is anchored to a terrestrial reference frame (TRF) that is defined within a conventional terrestrial reference system (CTRS). A CTRS has its origin at the Earth's center of mass and co-rotates with the Earth in its diurnal rotation. In this data dictionary, the principal CTRS of interest is the WGS 84 CTRS. The WGS 84 CTRS follows the criteria outlined by the International Earth Rotation and Reference Systems Service (IERS) and is defined as:

- a. Origin - Earth's center of mass
- b. Z-axis - The direction of the IERS Reference Pole (IRP)
- c. X-axis - Intersection of the IERS Reference Meridian (IRM) and the plane passing through the origin to the Z-axis
- d. Y-axis - Completes a system of right-handed orthonormal axes

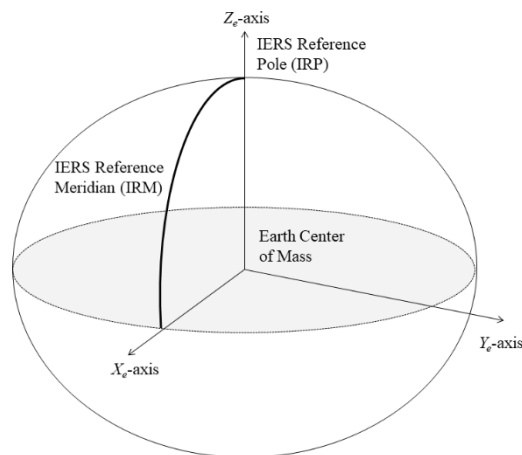


**Figure 2 - WGS 84 CTRS**

A TRF is a realization of a CTRS based on a set of observations and on alignments with international conventions. At the time of writing, the current realization of the WGS 84 CTRS is WGS 84 (G2296) as defined in NGA.STND.0036\_1.0.0\_WGS84. The complete list of realizations of WGS 84 CTRS at the time of writing is provided in Table 23, noting that further realizations are anticipated. Note that G2296 is aligned with the IERS International Terrestrial Reference Frame ITRF2020.

**Table 23 - WGS 84 reference frames**

| NAME           | IMPLEMENTATION DATE  |                       | EPOCH  |
|----------------|----------------------|-----------------------|--------|
|                | GPS BROADCAST ORBITS | NGA PRECISE EPHEMERIS |        |
| WGS 84         | 1987                 | 1 Jan 1987            |        |
| WGS 84 (G730)  | 29 Jun 1994          | 2 Jan 1994            | 1994.0 |
| WGS 84 (G873)  | 29 Jan 1997          | 29 Sep 1996           | 1997.0 |
| WGS 84 (G1150) | 20 Jan 2002          | 20 Jan 2002           | 2001.0 |
| WGS 84 (G1674) | 8 Feb 2012           | 7 May 2012            | 2005.0 |
| WGS 84 (G1762) | 16 Oct 2013          | 16 Oct 2013           | 2005.0 |
| WGS 84 (G2139) | 28 Mar 2021          | 3 Jan 2021            | 2016.0 |
| WGS 84 (G2296) | 7 Jan 2024           | 7 Jan 2024            | 2024.0 |

In this data dictionary, the following classes of TRF CRS are considered:

- a. Earth-centered, Earth-fixed (ECEF) CRS
- b. Geodetic ellipsoid CRS
- c. Geoid CRS
- d. Map projection CRS

The first two classes of CRS are three-dimensional coordinate systems tied to a given TRF. A geodetic ellipsoid CRS describes a position in terms of longitude, geodetic latitude, and ellipsoidal height. The principal geodetic ellipsoid CRS of interest is the WGS 84 Ellipsoid.

In common practice, a geoid CRS is based on a given geodetic ellipsoid CRS in which, at a given longitude and geodetic latitude, orthometric height - usually taken as mean sea level (MSL) height - is expressed as a geoid undulation above or below ellipsoidal height. The principal geoid CRS of interest is the WGS 84 Ellipsoid partnered with geoid undulations defined the WGS 84 EGM2008 Geoid.

A map projection CRS is the projection of a geodetic ellipsoid CRS onto a two-dimensional surface for a given longitudinal and latitudinal zone. The map projection CRS is considered historical and is discouraged in this data dictionary. An example of a map projection CRS would be the Universal Transverse Mercator (UTM) projection.

### 3.9.1.1 Earth-Centered, Earth-Fixed (ECEF) CRS

An Earth-centered, Earth-fixed (ECEF) CRS is a Cartesian CS that is anchored to given TRF such that the X-axis is simply the CTRS Xe-axis and so on. The coordinate tuple for the ECEF position vector is:

$$\vec{r} = (x_e, y_e, z_e)$$

Other quantity vectors may be referenced to the ECEF CRS as required.

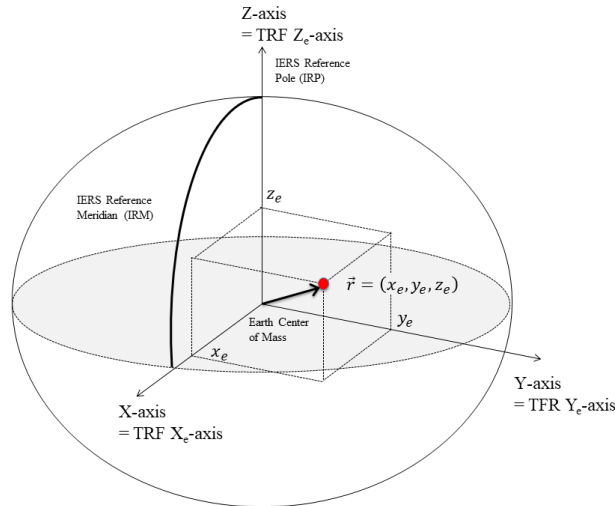


**Figure 3 - Earth-centered, Earth-fixed (ECEF) CRS**

### 3.9.1.2 Geodetic Ellipsoid CRS

The intended application of the geodetic ellipsoid CRS is geodetic positioning. A geodetic ellipsoid CRS is an ellipsoidal CS that is anchored to a TRF and has defining parameters that approximate to the size and shape of the Earth. The defining parameters of the WGS 84 Ellipsoid that are of interest to a CRS are:

- Length of semi-major axis,  $a_e = 6378137.0$  m
- Inverse flattening  $1/f_e = 298.257223563$

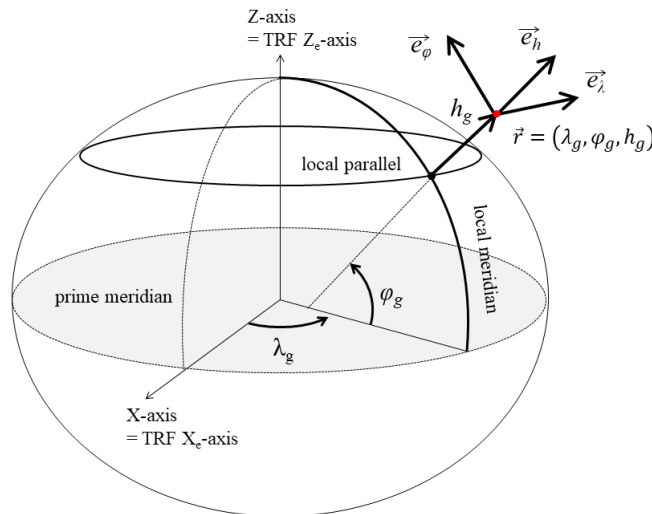


**Figure 4 - Geodetic ellipsoid CRS**

The coordinate tuple for the geodetic ellipsoid position vector is denoted by:

$$\vec{r} = (\lambda_g, \varphi_g, h_g)$$

When both the geodetic ellipsoidal CRS and ECEF CRS are anchored to the same terrestrial reference frame, then the coordinate conversion is:

$$x_e = (\rho + h_g) \cos \varphi_g \cos \lambda_g$$

$$y_e = (\rho + h_g) \cos \varphi_g \sin \lambda_g$$

$$z_e = \{(1 - e_e^2)\rho + h_g\} \sin \varphi_g$$

where the parameters  $\rho$  and  $e_e$  are defined in Table 22.

### 3.9.1.3 Geoid CRS

A geoid CRS is based on a geodetic ellipsoidal CRS except that ellipsoidal height,  $h_g$ , at a given point  $\vec{r} = (\lambda_g, \varphi_g, h_g)$  is replaced with orthometric height,  $H_{geoid}$ .

$$\vec{r} = (\lambda_g, \varphi_g, H_{geoid})_{geoid}$$

Orthometric height is the height of a body above or below the geoid rather than the height above or below the ellipsoid. The geoid serves as the vertical reference surface for Mean Sea Level (MSL). For WGS 84 based measurements, it is common practice to define the geoid at a point in terms of the distance above ( $+N_{geoid}$ ) or below ( $-N_{geoid}$ ) the WGS 84 Ellipsoid such that:

$$H_{geoid} = h_g - N_{geoid}$$

The current geoid model is WGS 84 EGM2008. Several data products are available for WGS 84 EGM2008 undulations, including undulation grid files and mathematical models.

## 3.9.2 Earth-Fixed Local Reference Frame

The second class of CRS is anchored to an Earth-fixed local reference frame (LRF) CD. There are two subtypes, which are Earth-fixed Local Tangent Plane and Earth Stationary Arbitrary Space.

### 3.9.2.1 Earth-Fixed Local Tangent Plane

The axes of the Earth-fixed local tangent plane (LTP) are determined by the unit vectors of the position differential of a geodetic ellipsoid CRS (see Figure 4). The orthonormal unit vectors  $\vec{e}_\lambda$ ,  $\vec{e}_\varphi$ , and  $\vec{e}_h$  define the LTP axes, where:

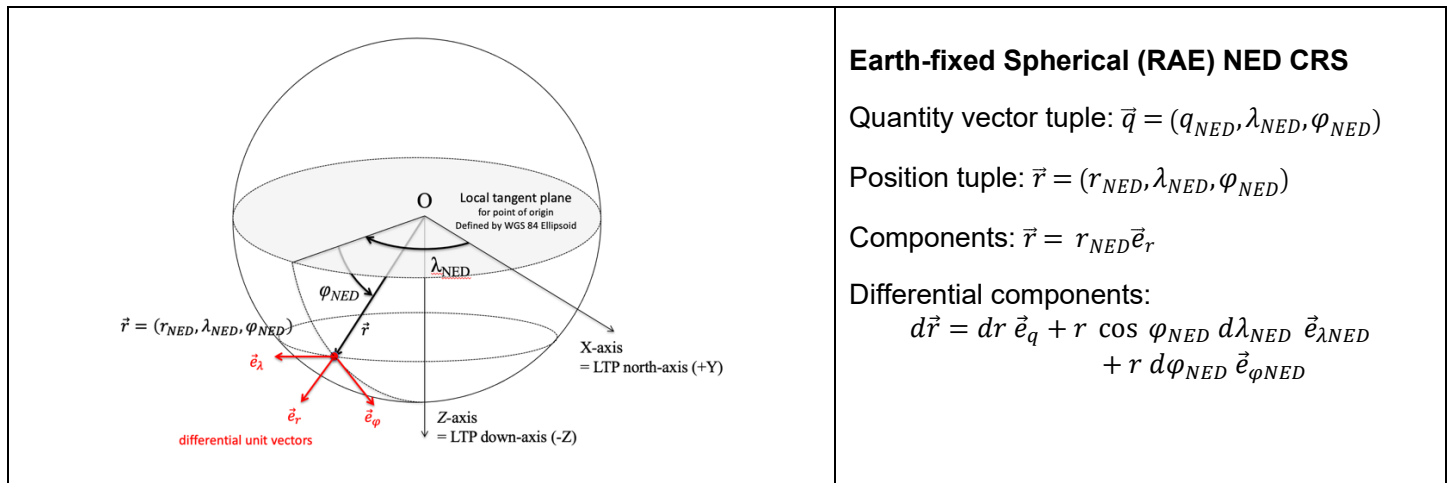
- $\vec{e}_\lambda$  is tangential to the local parallel eastward and defines the east-axis (E) (X-axis). In the LTP, this is known as the unit vector  $\vec{e}_E$ .
- $\vec{e}_\varphi$  is tangential to the local meridian northward and defines the north-axis (N) (Y-axis). In the LTP, this is known as the unit vector  $\vec{e}_N$ .
- $\vec{e}_h$  completes the right-handed system and defines the up axis (U) (Z-axis). In the LTP, this is known as the unit vector  $\vec{e}_U$ . Note that  $\vec{e}_D = -\vec{e}_U$ .

The origin of the LTP coordinate datum is an arbitrarily chosen Earth-fixed point. The Earth-fixed LTP may be used for local Earth positioning and as the basis for geo-referenced vector quantities. Table 24 shows four examples of coordinate reference systems that are anchored to the Earth-fixed local tangent plane:

- East-north-up (ENU)
- North-east-down (NED)
- Cylindrical
- Spherical range-elevation-azimuth (RAE)

Table 24 - Earth-fixed LTP CRS

|  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|--|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  | <p><b>Earth-fixed ENU CRS</b></p> <p>Quantity vector tuple: <math>\vec{q} = (q_E, q_N, q_U)</math></p> <p>Position tuple: <math>\vec{r} = (x_E, y_N, z_U)</math></p> <p>Components: <math>\vec{r} = x_E \vec{e}_E + y_N \vec{e}_N + z_U \vec{e}_U</math></p> <p>Differential components:<br/> <math display="block">d\vec{r} = dx_E \vec{e}_E + dy_N \vec{e}_N + dz_U \vec{e}_U</math></p>                                                                                             |
|  | <p><b>Earth-fixed NED CRS</b></p> <p>Quantity vector tuple: <math>\vec{q} = (q_N, q_E, q_D)</math></p> <p>Position tuple: <math>\vec{r} = (x_N, y_E, z_D)</math></p> <p>Components: <math>\vec{r} = x_N \vec{e}_N + y_E \vec{e}_E + z_D \vec{e}_D</math></p> <p>Differential components:<br/> <math display="block">d\vec{r} = dx_N \vec{e}_N + dy_E \vec{e}_E + dz_D \vec{e}_D</math></p>                                                                                             |
|  | <p><b>Earth-fixed Cylindrical NED CRS</b></p> <p>Quantity vector tuple: <math>\vec{q} = (q_{NE}, \lambda_{NE}, q_D)</math></p> <p>Position tuple: <math>\vec{r} = (\rho_{NED}, \lambda_{NED}, z_{NED})</math></p> <p>Components: <math>\vec{r} = \rho_{NED} \vec{e}_\rho + z_{NED} \vec{e}_D</math></p> <p>Differential components:<br/> <math display="block">d\vec{r} = d\rho_{NED} \vec{e}_\rho + \rho_{NED} d\lambda_{NED} \vec{e}_{\lambda_{NED}} + dz_{NED} \vec{e}_D</math></p> |



The cylindrical and spherical systems follow the NED convention, where the primary axis of rotation is anchored to the down-axis ( $\vec{e}_D$ ). This results in the azimuth being in a clockwise direction from north when projected on a map.

### 3.9.2.2 Earth-Stational Arbitrary Space Reference

An Earth-stationary arbitrary space CRS differs from an Earth-fixed LTP CRS in that its orthonormal axes are arbitrarily chosen instead of being fixed to the local tangent plane. These axes might be zeroed to the orientation of the vehicle body at some epoch or chosen according to some other criterion.

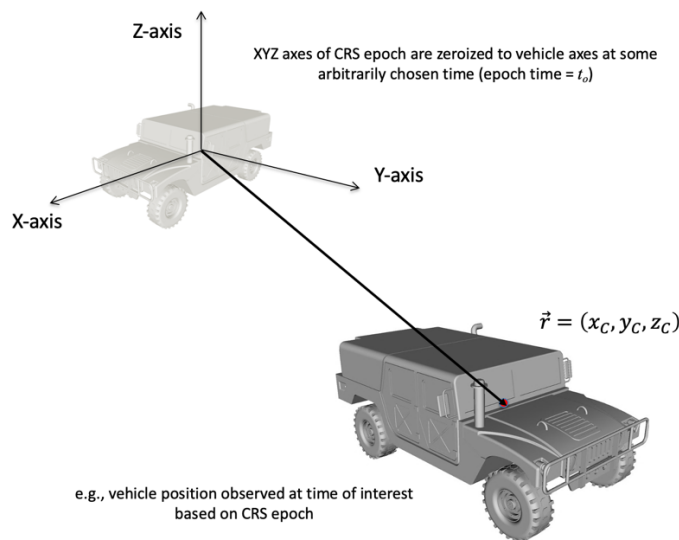


**Figure 5 - Example of Earth-stationary arbitrary XYZ CRS**

### 3.9.3 Body Kinematics LRF CRS

A body kinematics LRF CRS differs from the Earth-fixed LRF CRS in that the origin follows the position of the body rather than being an Earth-fixed point. However, at the time of interest, the origin is momentarily Earth-stationary with zero velocity and acceleration.

As with an Earth-fixed LRF, there are two subtypes. The first is oriented to the body's local LTP, and the second is oriented toward the object's own axes.

### 3.9.3.1 Body-Carried LTP CRS

An important application of this CRS is to measure a body's kinematics related to the local ENU or NED unit vectors. In some aerospace applications, it is convenient to establish a free-fall origin that is momentarily stationary but subject to the local acceleration of free fall. To cater for this requirement, the body-carried LTP CRS includes additional acceleration coordinates based on a free-fall reference.

### 3.9.3.2 Body-Carried, Body-Oriented CRS

This CRS subtype is based on the (momentarily stationary) axes of the body. For example, a vehicle's forward, right, and down axes (FRD) or a set of arbitrarily chosen X, Y, and Z axes. A vehicle's forward velocity (or speed) can be described in this CRS.

## 3.9.4 Celestial CRS

Celestial CRS are used in space operations. The CRS can be anchored to an Earth-centered inertial (ECI) CD or to a CD based on the center of mass (barycenter) of the solar system and the orientation of ECI axes at an arbitrarily defined epoch.

### 3.9.4.1 ECI

An example of an Earth-centered inertial CD is the J2000 reference frame, whose axes are defined by the mean equator and mean vernal equinox at the J2000.0 epoch. Unlike a TRF, the axes do not rotate with the Earth but are fixed relative to the celestial background.

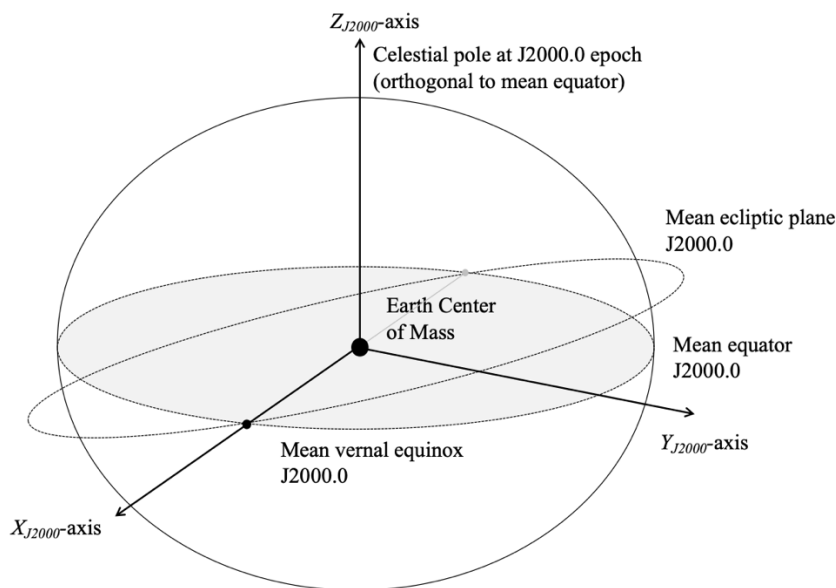


**Figure 6 - J2000 CD**

### 3.9.4.2 Barycentric

An example of a Barycentric CD is ICRF3. The ICRF3 is an inertial reference frame whose origin is the barycenter of the solar system and whose orientation is fixed by a catalog of observed extragalactic objects. The ICRF3 is the datum for the Barycentric Celestial Reference System (BCRS) and also the International Celestial Reference System (ICRS) via the geometric tensor  $G_{ab}$ . The ICRS-aligned axes are nominally orientated with the J2000 equatorial plane (X-Y) and vernal equinox (X) but not exactly because of measurement differences.

### 3.9.5 Engineering Body CRS

The final class of CRS is a coordinate system that is anchored to an engineering body. The most important subtype is vehicle body. The exact point of origin and axis orientation of the CD relative to the engineering body will be design specific. However, for vehicles, the axis labels will follow the following convention.

- a. Forward-axis (+X) - tied to the fuselage reference line through the point of origin in the direction of forward travel.
  - b. Right-axis (+Y) - assuming the vehicle has a symmetrical plane, the axis through the point of origin that is orthogonal to the symmetrical plane in the direction of right when facing forward (i.e., starboard).
  - c. Body down-axis (+Z) - the axis that completes the right-handed orthonormal system. It passes through the point of origin in the symmetrical plane at a right angle to the fuselage reference line in the direction of body down.
- a. If the vehicle has a navigation system, then the navigation system will define the reference for the CD.

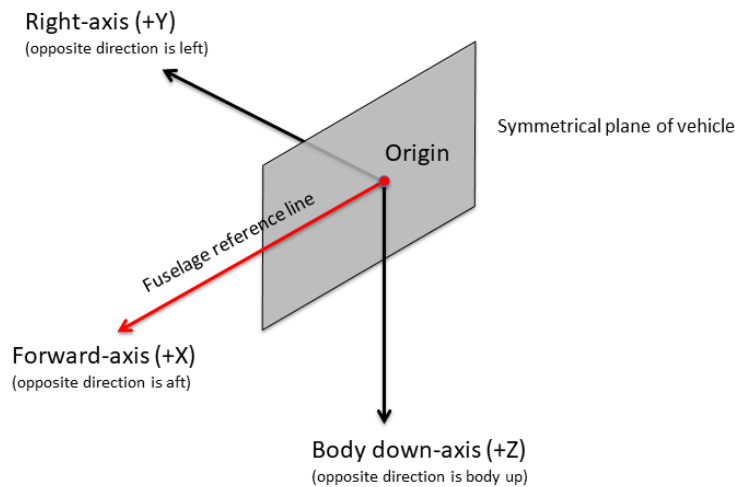


**Figure 7 - Vehicle body CD or payload body CD**

Figure 8 is an example of a vehicle body CD applied to an aircraft.

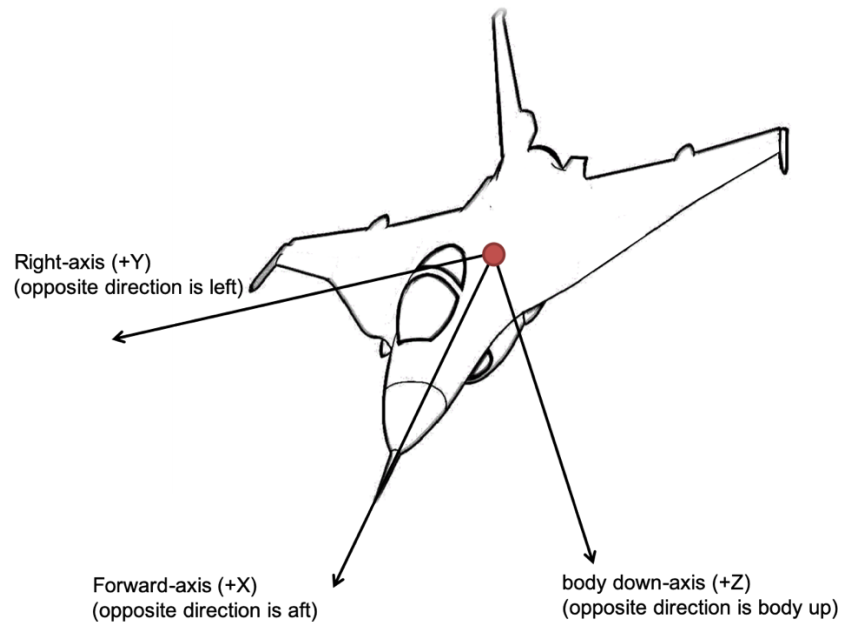
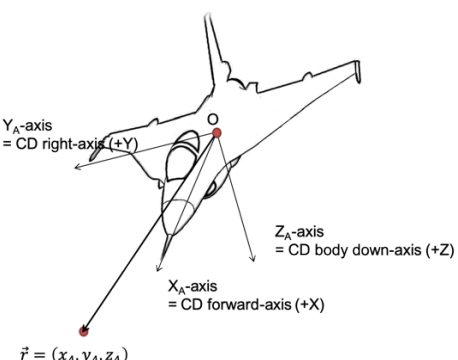
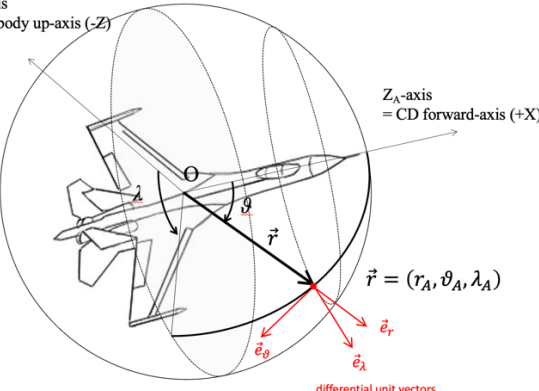
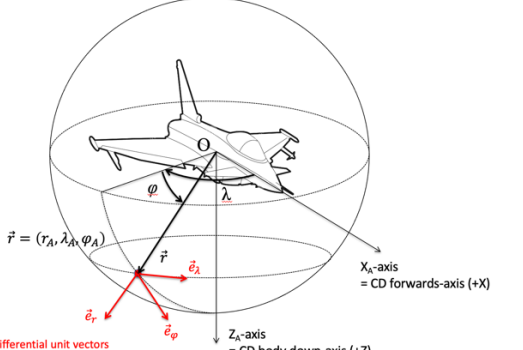


**Figure 8 - Vehicle body CD applied to aircraft**

Table 25 shows various CRS that are anchored to a vehicle body coordinate datum. The vehicle body Cartesian CRS anchors the Cartesian CS X-axis to the CD forward-axis (+X), the CS Y-axis to the CD right-axis (+Y), and the CS Z-axis to the CD down-axis (+Z). There are two spherical CRS. The first anchors the CD Z-axis to the CD forward-axis (+X) and the CD X-axis to the CD up-axis (-Z). In contrast, the spherical (RAE) system anchors the CD Z-axis to the CD down-axis (+Z) and the CS X-axis to the CD forward-axis (+X).

**Table 25 - Example vehicle body CRS**

|                                                                                                                                                                                 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|  <p><math>\vec{r} = (x_A, y_A, z_A)</math></p>                                                 | <p><b>Vehicle Body Cartesian CRS</b></p> <p>Quantity vector tuple: <math>\vec{q} = (q_{Ax}, q_{Ay}, q_{Az})</math></p> <p>Position tuple: <math>\vec{r} = (x_A, y_A, z_A)</math></p> <p>Components: <math>\vec{r} = x_A \vec{e}_x + y_A \vec{e}_y + z_A \vec{e}_z</math></p> <p>Differential components:<br/> <math display="block">d\vec{r} = dx_A \vec{e}_x + dy_A \vec{e}_y + dz_A \vec{e}_z</math></p>                                                                |
|  <p><math>\vec{r} = (r_A, \vartheta_A, \lambda_A)</math></p> <p>differential unit vectors</p> | <p><b>Vehicle Body Spherical CRS</b></p> <p>Quantity vector tuple: <math>\vec{q} = (q_A, \vartheta_A, \lambda_A)</math></p> <p>Position tuple: <math>\vec{r} = (r_A, \vartheta_A, \lambda_A)</math></p> <p>Components:<br/> <math display="block">\vec{r} = r_A \vec{e}_r</math></p> <p>Differential components:<br/> <math display="block">d\vec{r} = dr \vec{e}_r + r d\vartheta_A \vec{e}_{\vartheta_A} + r \sin \vartheta_A d\lambda_A \vec{e}_{\lambda_A}</math></p> |
|  <p><math>\vec{r} = (r_A, \lambda_A, \varphi_A)</math></p> <p>differential unit vectors</p>  | <p><b>Vehicle Body Spherical (RAE) CRS</b></p> <p>Quantity vector tuple: <math>\vec{q} = (q_A, \lambda_A, \varphi_A)</math></p> <p>Position tuple: <math>\vec{r} = (r_A, \lambda_A, \varphi_A)</math></p> <p>Components: <math>\vec{r} = r \vec{e}_r</math></p> <p>Differential components:<br/> <math display="block">d\vec{r} = dr \vec{e}_r + r \cos \varphi_A d\lambda_A \vec{e}_{\lambda_A} + r d\varphi_A \vec{e}_{\varphi_A}</math></p>                            |

### 3.10 Coordinate Operations

In this data dictionary, coordinate operations re-reference a quantity from one CRS to another. The value of the quantity itself does not change. Coordinate operations are different from vector operations. In a vector operation, the CRS would remain the same and the vector would be operated on within that CRS (for example, by a rotation matrix).

It is not the purpose of this data dictionary to define normative coordinate operations between every CRS nor to define algorithms for software. However, many parameters of coordinate operations are exchanged between systems and, therefore, a notional coordinate operation reference system is required.

A hypothetical coordinate operation from CRS1 to CRS2 involves three basic stages:

- Conversion of CRS1 from its “coordinate form” into its “datum form.”
- Transformation of the CRS1 CD into the CRS2 CD.
- Conversion of CRS2 from its “datum form” to its “coordinate form.”