

Manual on Design and Manufacture of Torsion Bar Springs - SAE J796a

Report of Spring Committee approved August 1947 and last
revised August 1966.

SAE Information Report
First edition published 1947
Second edition published 1966

Published by:
Society of Automotive Engineers, Inc., 485 Lexington Ave., New York, N. Y. 10017



Documents

Document Sharing Hub

© Society of Automotive Engineers, Inc., 1966

All technical reports, including standards approved and practices recommended, are advisory only. Their use by anyone engaged in industry or trade is entirely voluntary. There is no agreement to adhere to any SAE Standard or SAE Recommended Practice, and no commitment to conform to or be guided by any technical report.

In formulating and approving technical reports, the Technical Board, its Councils and Committees will not investigate or consider patents which may apply to the subject matter. Prospective users of the report are responsible for protecting themselves against liability for infringement of patents.

—SAE Technical Board Rules and Regulations

Preface

The 1966 edition of this Manual supersedes the original edition which was published in 1947. The scope of the work has been considerably expanded to reflect the wide range of experience which has been accumulated in many fields of torsion bar application during the intervening years.

While the Manual has not been changed in its basic structure, it has been updated with the inclusion of hexagonal end connections, of hollow and laminated torsion bars, of the arc type transition section, and of stabilizer bars. A new section deals with fatigue testing and the life expectancy of torsion bar springs in relation to laboratory test data.

The Spring Committee hopes that the Manual, in this new form, will prove even more valuable to torsion bar designers, manufacturers, and users, than the original edition which met with universal approval. The Committee wishes to note -- as it previously has done upon presenting its other Spring Manuals -- that the text should not be regarded as a compilation of design or manufacturing specifications. Rather, the Manual is a reference work which conveys essential information on solving the problems encountered with torsion bar springs.

The Spring Committee recognizes that time and effort have been spent in generous measure by the members of the Subcommittee who were selected for writing and editing the revised Manual. Thanks are herewith extended by the Committee.

Spring Committee, 1965

F. R. NAIL (Sponsor), Engineering Div., Mack Trucks, Inc.; BERNHARD STERNE (Chairman), Bernhard Sterne Associates; J. T. HOBAN (Vice Chairman), Cadillac Motor Car Div., General Motors Corp.; G. N. BERGUM, Defense Operations Div., Chrysler Corp.; H. H. CLARK, Spring Div., Eaton Yale & Towne Inc.; H. J. ELMENDORF, U. S. Steel Corp.; L. C. ELDER, Wallace Barnes Co. Ltd.; R. C. EMIG, Chevrolet Motor Div., General Motors Corp.; D. F. FISCUS, Engineering & Research Staff, Ford Motor Co.; R. W. HILL, Buick Motor Div., General Motors Corp.; K. KUHN, Rockwell-Standard Corp.; K. W. MAIER, Math + Metrik, Inc.; G. F. MEYER, International Harvester Co.; G. N. MYRICK, U. S. Army Tank-Automotive Center; J. K. NEMETH, Auto Div., American Motors Corp.; J. F. O'NEIL, U. S. Army Springfield Armory; B. S. PARKER, U. S. Army Watertown Arsenal; G. L. RADAMAKER, Spring Div., Eaton Yale & Towne, Inc.; J. W. ROSENKRANDS, Engineering Staff, General Motors Corp.; F. T. ROWLAND, Leaf Spring Div., Rockwell Standard Corp.; J. E. SILVIS, American Steel Foundries, Inc.; W. WATSON, U. S. Army Edgewood Arsenal; J. W. WELLS, Associated Spring Corp.; W. M. WOOD, Barnes-Gibson-Raymond Div., Associated Spring Corp.

Torsion Bar Spring Subcommittee, 1965

G. N. BERGUM (Chairman), Defense Operations Div., Chrysler Corp.; K. C. ADAMS, JR., Farm Equipment Div., Allis-Chalmers Manufacturing Co.; K. ARNING, Ford Motor Co.; S. BACKAITIS, Chrysler Corp.; W. F. BRANDT, U. S. Army Tank-Automotive Center; J. A. CALDWELL, U. S. Army Engineer Research & Development Labs.; LYLE EATON, LeTourneau-Westinghouse Co.; J. S. EDGAR, Ontario Steel Products Co. Ltd.; E. J. GOW, JR., U. S. Army Tank-Automotive Center; R. E. HANSLIP, Mather Co.; N. E. HENDRICKSON, Consulting Engineer, Laguna Beach, Calif.; W. T. HUNT, Weapons Command, U. S. Army Rock Island Arsenal; T. E. KASHMERICK, Chevrolet Motor Div., General Motors Corp.; D. M. KELLEY, Euclid Div., General Motors Corp.; C. J. KROPF, U. S. Army Tank-Automotive Center; W. H. McCORMICK, Crucible Steel Co. of America; E. F. OAKWOOD, U. S. Army Tank-Automotive Center; G. L. RADAMAKER, Spring Div., Eaton Yale & Towne Inc.; MAX RUEGG, Detroit, Mich.; C. A. TEA, Ford Div., Ford Motor Co.; R. W. WOLF, American Steel & Wire Div., U. S. Steel Corp.; W. J. YOUNG, White Motor Corp.

Table of Contents

CHAPTER	Page
I INTRODUCTION	1
II DESIGN CALCULATIONS	2
1. Symbols Used in Formulae	2
2. Round Bar in Torsion	2
3. Rectangular Bar in Torsion	2
A. Straight Bar of Solid Rectangular Cross-Section	2
B. Straight Laminated Bar	3
4. Torsion Bar Spring and Lever	4
5. Sample Computation	8
A. Requirements for Round Bar	8
B. Computation for Round Bar	8
C. Consideration for Laminated Bar	9
D. Computation for Laminated Bar of Square Cross-Section	9
E. Computation for Laminated Bar with Nearly Circular Cross-Section	10
F. Solutions for Rate Correction of Laminated Bar	10
6. Operating Stresses	13
A. Suspension Springs	13
B. Other Torsion Bar Springs	14
III DESIGN OF END FASTENING	15
1. End Configuration	15
A. Serrated End Connection	15
B. Hexagonal End Connection	15
2. Anchor Member	15
A. Serrated Anchor	15
B. Hexagonal Anchor	16
3. Transition Section	16
IV CONTROL OF ASSEMBLY POSITION	19
1. Methods to Insure Correct Vehicle Height:	19
A. Infinitesimal Adjustment by Screw	19
B. Adjustment by Vernier Steps	20
C. Equal Serrations at Both Ends	20
D. Blocked Serrations at Both Ends	20
2. Marking of Windup Direction	20

V	MATERIAL AND PROCESSING	21
	1. General Requirements	21
	A. Material.....	21
	B. Value of Shear Modulus.....	21
	C. Upsetting and Machining	21
	D. Heat Treatment	22
	E. Straightening.....	22
	F. Shot Peening	22
	G. Presetting.....	22
	H. Corrosion Protection.....	23
	I. Special Testing.....	23
	2. Presetting	23
	A. Load Deflection Curve.....	23
	B. Stress Distribution	24
	C. Selection of Preset Strain	24
	3. Fatigue Testing	25
	A. Choice of Test Conditions	26
	B. Frequency Distribution of Test Data.....	26
	C. Statistical Inference from Test Sample to Population.....	27
	D. Observations	27
	E. Literature References	28
VI	TORSION BAR SPRING APPLICATIONS	29
	1. Current Design Examples:	29
	A. Square Torsion Bar for Boat Trailer	29
	B. Hexagonal Torsion Bar for Truck Tilt Cab	30
	C. Laminated Torsion Bar for Truck Tilt Cab	31
	2. Stabilizer Bar	33
	A. Design Considerations	33
	B. Design Example	34

Chapter I — Introduction

Torsion bar springs are used a wide range of installations, from precision instruments through balance springs to automotive and military tank suspension springs. The designs vary widely in efficiency of material utilization (foot-pound of stored energy per unit volume of spring material), in complexity, and in production costs. This Manual presents information particularly pertinent to high efficiency springs as used on suspensions for automotive vehicles. It is applicable, however, to many types of torsion bar springs for a wide variety of uses. Much of the information on which the Manual is based was developed during work on World War II ordnance contracts in the years 1942-1945. The Manual has been revised to reflect up-to-date designs and manufacturing processed.

Chapter II — Design Calculations

1. SYMBOLS USED IN FORMULAE

P	= Force at end of lever, perpendicular to reference line (Fig. 2), lb
P_{static}	= Static load (used for P in suspension computations), lb
α	= Angle between reference line and lever when P is applied; positive as shown in Fig. 2, radians
α_{static}	= Angle α when P_{static} is applied; positive as shown in Fig. 2, radians
β	= Angle between reference line and lever at zero load position; positive as shown in Fig. 6, radians
θ	= Windup angle when P is applied ($= \alpha + \beta$), radians
θ_{static}	= Windup angle when P_{static} is applied ($= \alpha_{\text{static}} + \beta$) (Fig. 6), radians
θ_{comp}	= Windup angle when the maximum operating load is applied, with the lever in compression (jounce) position ($= \alpha_{\text{comp}} + \beta$) (Fig. 6), radians
d	= Diameter of round bar (outside diameter of hollow bar), in.
d_i	= Inside diameter of hollow bar, in.
t	= Thickness of rectangular bar or square bar, in.
w	= Width of rectangular bar (long side of cross section), in.
L	= Active length of bar (defined in Chapter III, Section 3), in.
R	= Length of lever (Fig. 2), in.
f	= Deflection at end of lever from reference line when P is applied, measured parallel to direction of P ($= R \sin \alpha$); positive as shown in Fig. 2, in.
f_a	= Deflection at end of lever from reference line when P_{static} is applied, measured parallel to direction of P_{static} ($= R \sin \alpha_{\text{static}}$); positive as shown in Fig. 6, in.
f_b	= Deflection at end of lever from zero load position to reference line, measured parallel to direction of P ($= R \sin \beta$), in.
$f_a + f_b$	= Total static deflection at end of lever, from zero load position to static load position, measured parallel to direction of P_{static} , in.
δ	= Effective static deflection at end of lever, equal to load divided by spring rate prevailing at that load, in.
T	= Torque applied to bar ($= P R \cos \alpha$), lb-in.
k	= Spring rate at end of lever (variable), measured parallel to direction of P , lb/in.

k_T	= Torsional spring rate of bar ($= T/\theta$), lb-in./radian
γ	= Shear strain
G	= Shear modulus (see Chapter V, Section 1), psi
S_s	= Shear stress, psi
η_2	= Saint Venant's stress coefficient (Fig. 1)
η_3	= Saint Venant's stiffness coefficient (Fig. 1)
C_1	= Load factor
C_2	= Rate factor
C_3	= Static deflection factor
N	= Number of laminae in laminated bar

defined in
Chapter II,
Section 4

2. ROUND BAR IN TORSION

Table 1 shows the relations which exist for the following conditions:

- Bar is straight.
- Uniform cross section of solid or hollow circular configuration.
- If hollow, inside and outside diameters are concentric.
- Torsionally loaded only.

3. RECTANGULAR BAR IN TORSION

A. STRAIGHT BAR OF SOLID RECTANGULAR CROSS SECTION - For a straight bar of solid rectangular cross section, loaded in torsion only, the following relations exist:

Windup angle (radians):

$$\theta = \frac{T L}{\eta_3 t^3 w G} = \frac{\eta_2 S_s L}{\eta_3 t G}$$

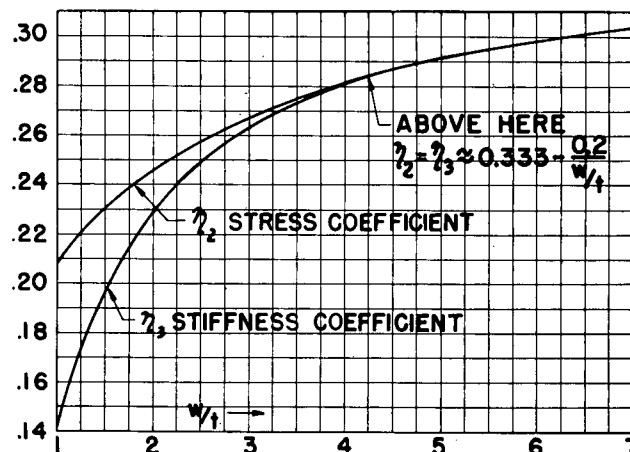


Fig. 1 - Saint Venant's coefficients for rectangular bar in torsion

Table 1 - Relations Existing for Round Bar in Torsion

Quantity	Symbol	Equivalent		Units
		Solid	Hollow	
Windup Angle	θ	$\frac{32 T L}{\pi d^4 G}$ $\frac{2 S_s L}{d G}$	$\frac{32 T L}{\pi d^4 G \left[1 - \left(d_i^4 / d^4 \right) \right]}$ $\frac{2 S_s L}{d G}$	radians
Torsional Rate	k_T	$\frac{T}{\theta}$ $\frac{\pi d^4 G}{32 L}$	$\frac{T}{\theta}$ $\frac{\pi d^4 G \left[1 - \left(d_i^4 / d^4 \right) \right]}{32 L}$	lb-in./radian
Stress	S_s	$\frac{16 T}{\pi d^3}$ $\frac{\theta d G}{2 L}$	$\frac{16 T}{\pi d^3 \left[1 - \left(d_i^4 / d^4 \right) \right]}$ $\frac{\theta d G}{2 L}$	psi
Stress Rate	$\frac{S_s}{\theta}$	$\frac{d G}{2 L}$	$\frac{d G}{2 L}$	psi/radian

Torsional rate (lb-in./radian):

$$k_T = \frac{T}{\theta} = \frac{\eta_3 t^3 w G}{L}$$

Stress (psi):

$$S_s = \frac{T}{\eta_2 t^2 w} = \frac{\eta_3 \theta t G}{\eta_2 L}$$

Stress rate (psi/radian):

$$\frac{S_s}{\theta} = \frac{\eta_3 t G}{\eta_2 L}$$

B. STRAIGHT LAMINATED BAR - For a straight laminated bar (each lamina of rectangular cross section, same material and same length), loaded in torsion only, the following relations exist when the bar has N_1 laminae of t_1

thickness, w_1 width, T_1 torque, $\eta_{2,1}$ and $\eta_{3,1}$ coefficients; also N_2 laminae of t_2 , w_2 , T_2 , $\eta_{2,2}$ and $\eta_{3,2}$; N_3 laminae....:

Windup angle:

$$\begin{aligned} \theta &= \frac{T_1 L}{\eta_{3,1} t_1^3 w_1 G} = \frac{\eta_{2,1} S_s L}{\eta_{3,1} t_1 G} \\ &= \frac{T_2 L}{\eta_{3,2} t_2^3 w_2 G} = \frac{\eta_{2,2} S_s L}{\eta_{3,2} t_2 G} \end{aligned}$$

and so on

Torsional rate:

$$k_T = \frac{T}{\theta} = \frac{N_1 T_1 + N_2 T_2 + \dots}{\theta}$$



Documents

Document Sharing Hub

Stress:

$$S_s = \frac{T_1}{\eta_{2,1} t_1^2 w_1} = \frac{\eta_{3,1} \theta t_1 G}{\eta_{2,1} L}$$

$$= \frac{T_2}{\eta_{2,2} t_2^2 w_2} = \frac{\eta_{3,2} \theta t_2 G}{\eta_{2,2} L}$$

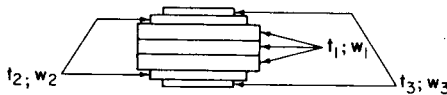
and so on

Stress rate:

$$\frac{S_s}{\theta} = \frac{\eta_{3,1} t_1 G}{\eta_{2,1} L} = \frac{\eta_{3,2} t_2 G}{\eta_{2,2} L}$$

and so on

In actual practice, the dimensions for a laminated torsion bar can be established by the following procedure, considering the case where the complete bar is to have a rate k_T and this cross section:



From given data the maximum windup angle (θ_{comp}) will be computed at which all laminae may be subjected to the maximum allowable stress ($S_s(comp)$). With all laminae having the same active length (L), the maximum allowable thickness for each lamina is

$$t_{max} = \frac{\eta_2}{\eta_3} \times \frac{S_s(comp) L}{\theta_{comp} G}$$

Any lamina thicker than t_{max} will be overstressed.

All laminae are wound up to the same θ_{comp} and would carry the same maximum stress if the thickness of all laminae could be held to t_{max} . This would result in the most efficient spring or in a spring of minimum weight. In most cases this is not practical and the thickness of the laminae must be adjusted to give the required rate. It should, however, be kept in mind that the stresses should be held as close as possible to the allowable maximum stress in order to keep the spring weight to a minimum.

From the desired spring configuration, the widths (w) and the number (N) of the laminae in each group may be tentatively determined; the rate equation in Section 1 of this chapter furnishes the torsional rate of one lamina in each

group:

$$k_{T,1} = \frac{\eta_{3,1} t_1^3 w_1 G}{L}$$

$$k_{T,2} = \frac{\eta_{3,2} t_2^3 w_2 G}{L}$$

and so on

(For first try, it will be best to make $t_1 = t_2 = t_n = t_{max}$)

Then the torsional rate of the complete bar is:

$$k_T = N_1 k_{T,1} + N_2 k_{T,2} + \dots + N_n k_{T,n}$$

In general, this result will not coincide with the desired value figured from

$$k_T = \frac{P R \cos \alpha}{\alpha + \beta}$$

See Chapter II, Section 4.

If the rate comes out too low, another lamina will have to be added, unless it is possible to make an adjustment in the widths of the laminae. (It is assumed that the length must be held.) If the rate comes out too high, thinner laminae must be used or the widths must be reduced.

It is important to remember that no laminae should exceed t_{max} , but all laminae should be kept as close as possible to this thickness. The stress is not affected by a change in width (w) as long as the ratio $\eta_3/\eta_2 = 1$; this holds true for most practical applications where $(w/t) > 3.5$.

Consequently, changing w is a convenient way of rate adjustment if the widths of the laminae are not required to be held to existing commercial sizes.

Changing the active length (L) of the bar, provided the specific design allows this, is also a means of rate adjustment; however, L also affects the stress, and the whole computation will have to be repeated.

4. TORSION BAR SPRING AND LEVER

Fig. 2 shows a combination of torsion bar spring and lever which is frequently used in suspensions. In this diagram the deflection f and the angles α and β are measured from a reference line which is perpendicular to the applied load and passes through the center of the torsion bar. They are counted positive when their relations to the reference line are as shown in Fig. 2.

The load deflection characteristics of this mechanism are

not linear but are given by the following:

$$P = \frac{T}{R \cos \alpha}$$

$$k_T = \frac{T}{\theta}$$

$$\theta = \alpha + \beta$$

$$k_T = \frac{T}{\alpha + \beta}$$

$$P = \frac{k_T (\alpha + \beta)}{R \cos \alpha} = \frac{k_T}{R} C_1 \left[C_1 = \frac{\alpha + \beta}{\cos \alpha} = \frac{\theta}{\cos \alpha} \right]$$

$$\frac{f}{R} = \sin \alpha$$

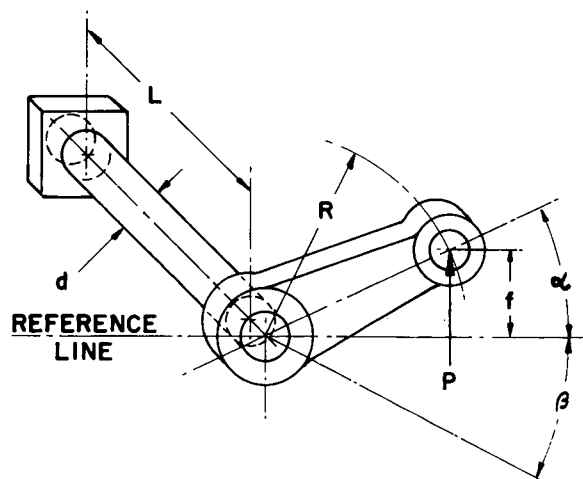


Fig. 2 - Torsion bar spring and lever

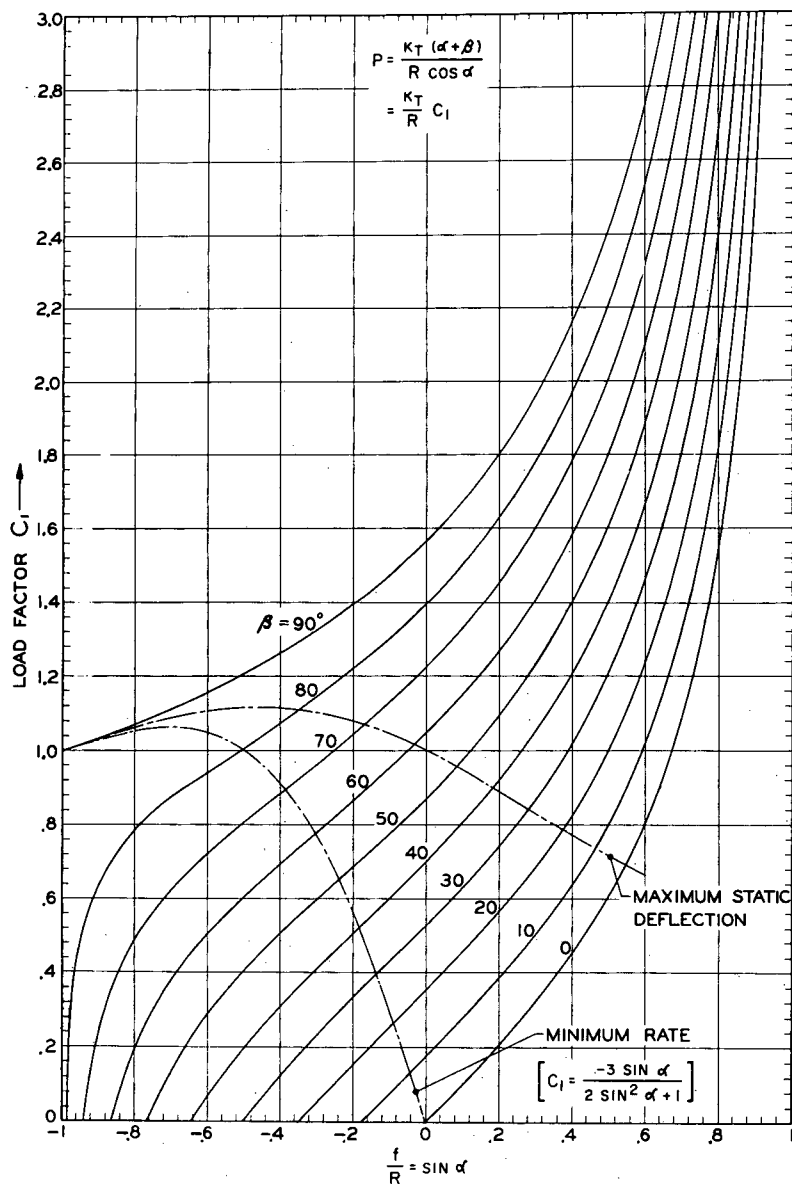


Fig. 3 - Load factor versus deflection

tion diagrams for the end of the lever.

The rate will be a minimum when dk/df is equal to zero, or when

$$C_1 = \frac{-3 \sin \alpha}{2 \sin^2 \alpha + 1}$$

It will be noted from the chart that C_1 becomes equal to zero when $\beta = -\alpha$.

The vertical rate at the end of the lever is given by $k = (dP/df)$. Using the value of P above and the relation $f = R \sin \alpha$

$$P = \frac{k_T}{R} \times \frac{(\alpha + \beta)}{\cos \alpha}$$

$$\frac{dP}{d\alpha} = \frac{k_T}{R} \left[(\alpha + \beta) \frac{\tan \alpha}{\cos \alpha} + \frac{1}{\cos \alpha} \right]$$

$$\frac{dP}{d\alpha} = \frac{k_T}{R} \left[\frac{1 + (\alpha + \beta) \tan \alpha}{\cos \alpha} \right]$$

$$f = R \sin \alpha$$

$$\frac{df}{d\alpha} = R \cos \alpha$$

$$k = \frac{dP}{df} = \frac{dP}{d\alpha} \cdot \frac{d\alpha}{df}$$

$$= \frac{k_T}{R^2} \times \frac{1 + (\alpha + \beta) \tan \alpha}{\cos^2 \alpha}$$

$$k = \frac{k_T}{R^2} C_2$$

$$C_2 = \frac{1 + (\alpha + \beta) \tan \alpha}{\cos^2 \alpha}$$

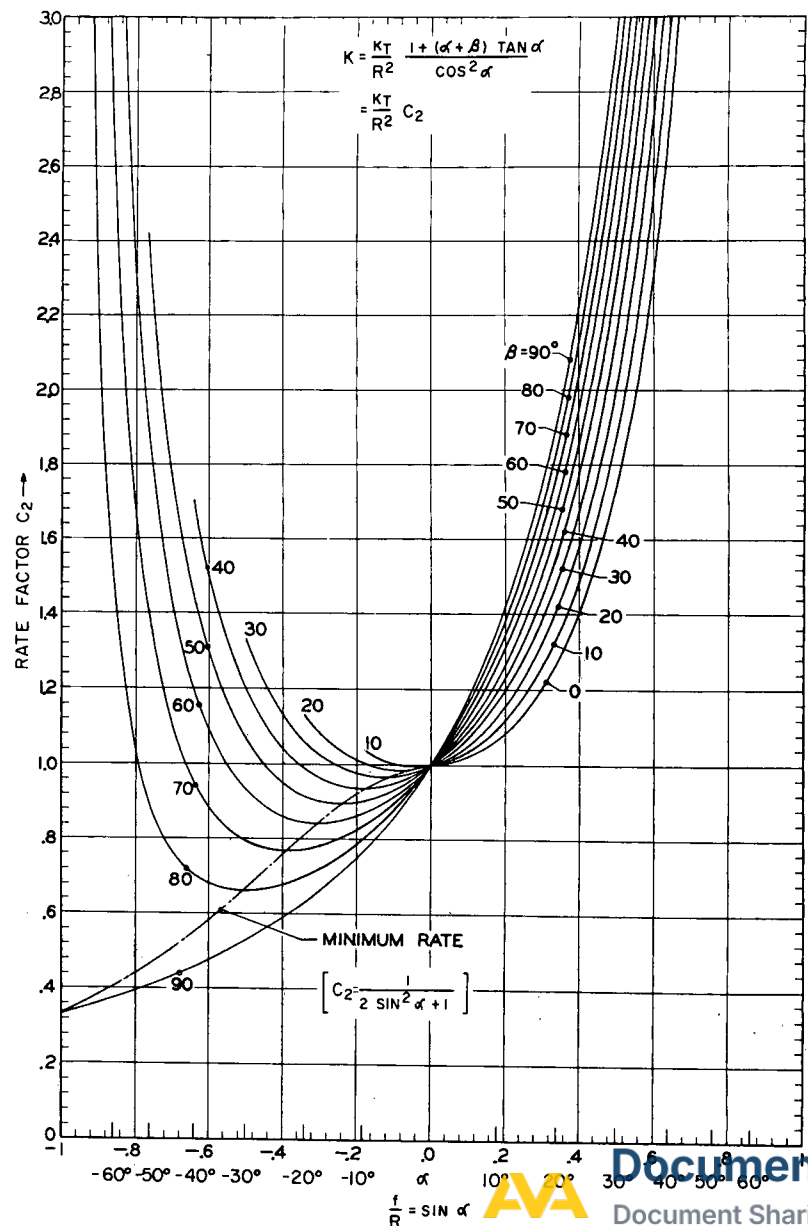


Fig. 4 - Rate factor versus deflection

Fig. 4 shows the function C_2 plotted against the ratio f/R or $\sin \alpha$ as abscissa. It shows that the rate of such a torsion bar and lever is by no means constant, and that the minimum rate occurs at a position of lever centerline below the horizontal. When f/R is zero then for all values of β , $C_2 = 1.0$.

The rate will be a minimum when dk/df is equal to zero, or when $C_2 = 1/(2 \sin^2 \alpha + 1)$ only for negative values of $\sin \alpha$.

The static deflection at any point is defined as $\delta = P/k$. Using the above values this becomes

$$\delta = R \frac{\cos \alpha}{\frac{1}{\alpha + \beta} + \tan \alpha}$$

$$\delta = R C_3$$

$$C_3 = \frac{\cos \alpha}{\frac{1}{\alpha + \beta} + \tan \alpha}$$

Fig. 5 shows the function C_3 plotted against the ratio f/R or $\sin \alpha$ as abscissa, so that the curves represent a comparison of static vertical deflection with the vertical deflection from the reference line. By definition, the abscissa values in Fig. 5 actually represent $\sin \alpha_{\text{static}}$ or f_a/R .

The rate will be a minimum when dk/df is equal to zero, or when $C_3 = -3 \sin \alpha$.

It is to be noted that the curves for minimum rate, maximum static deflection, and $\beta = 90^\circ$ all intersect at $f/R = -1.0$ or $C_3 = 3.0$.

NOTE: All angles in previous equations are expressed in radians. In Figs. 3-5 the angle β is given in degrees, but for calculation of loads and rates from the constants C_1 and C_2 the torsional rate must be expressed in the dimensions lb-in./radian. Where necessary, angles are converted from degrees to radians by the relation 1 radian = 57.296 deg as shown in the examples.

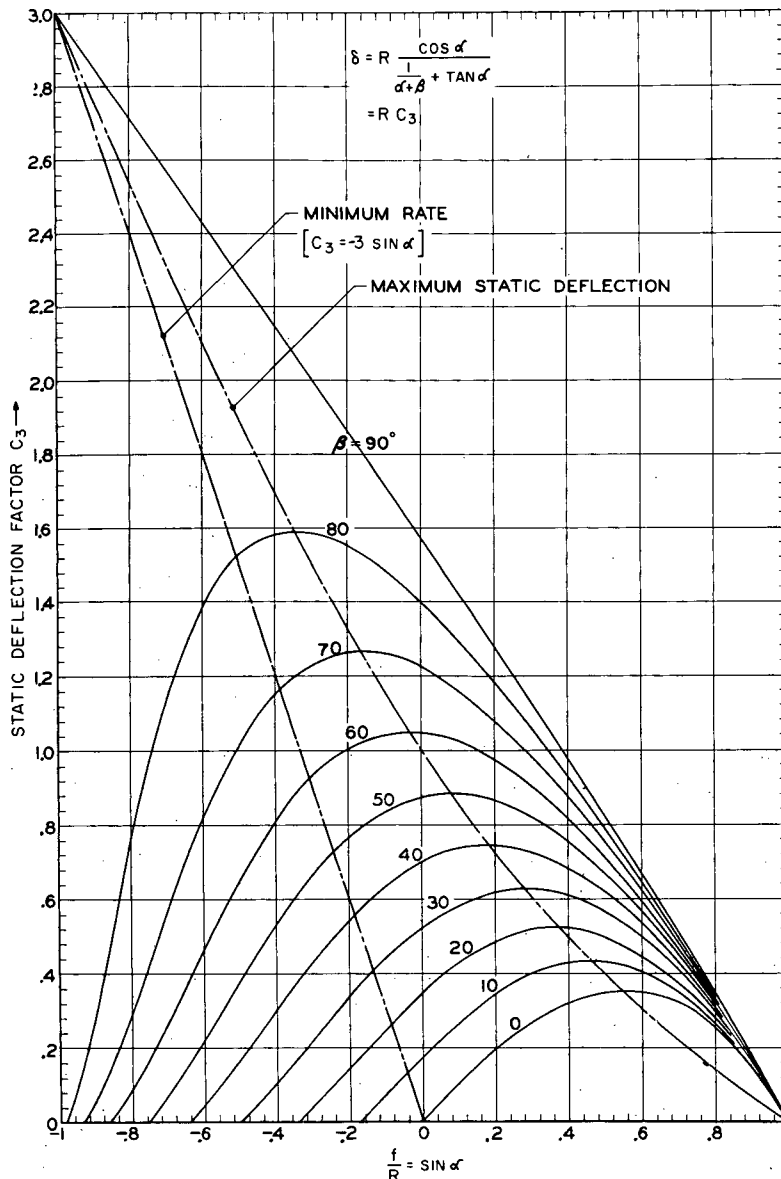


Fig. 5 - Static deflection factor versus deflection