

Methods of Residual Stress Measurement — SAE J936



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Methods of Residual Stress Measurement — SAE J936

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The derivation of equations on page 25-26 is incorrect. Starting with the last paragraph in the left column on page 25, the material should read:

Letting σ''_x equal the equivalent uniaxial stress that produces the strain ϵ_x , then ϵ_x equals σ''_x/E , and using similar reasoning, ϵ_y equals σ''_y/E . Making these substitutions, the biaxial stress are

$$\sigma_x = \frac{\sigma''_x + \mu \sigma''_y}{1 - \mu^2} \quad (4-10)$$

and

$$\sigma_y = \frac{\sigma''_y + \mu \sigma''_x}{1 - \mu^2} \quad (4-11)$$

Now σ''_x and σ''_y can be measured, using Eq. 4-4, where the curvature terms include the appropriate subscripts. Substituting Eq. 4-4, with the proper subscripts, into Eq. 4-10, we obtain

$$\begin{aligned} \sigma_x = \frac{E}{1 - \mu^2} & \left[\frac{(t - z)^2}{6} \left(\frac{dC_x}{dz} + \mu \frac{dC_y}{dz} \right) \right. \\ & - \frac{2}{3} (t - z) (C_x + \mu C_y) \\ & \left. + \frac{1}{3} \int_0^z (C_x + \mu C_y) dz \right] \end{aligned} \quad (4-12)$$

The equation for σ_y is exactly similar. The definition of terms and their sign conventions were given previously.

If identically the same approach is used with Eq. 4-6, the equation for the biaxial stress σ_x , using strain readings,

$$\begin{aligned} \sigma_x = \frac{E}{1 - \mu^2} & \left[+ \frac{t - z}{2} \left(\frac{d\epsilon_x}{dz} + \mu \frac{d\epsilon_y}{dz} \right) + 2 (\epsilon_x + \mu \epsilon_y) \right. \\ & \left. - 3 (t - z) \int_0^z \frac{\epsilon_x + \mu \epsilon_y}{(t - z)^2} dz \right] \end{aligned} \quad (4-13)$$

A similar expression may be written for σ_y .

(b) Secondary Layer Theory - The stresses may be measured by Eqs. 4-12 and 4-13, but they must be corrected for the change in stress produced by the primary cuts. These corrections are similar to Eqs. 4-7 and 4-8, and become

$$\begin{aligned} \sigma'_x - \sigma_x = \frac{E}{1 - \mu^2} & \left[- \frac{m}{3} (C'_x + \mu C'_y) \right. \\ & - \frac{1}{3} \int_0^{z=t-m} (C_x + \mu C_y) dz \\ & \left. + n (C'_x + \mu C'_y) \right] \end{aligned} \quad (4-14)$$

and

$$\begin{aligned} \sigma'_x - \sigma_x = \frac{E}{1 - \mu^2} & \left[+ \left(1 - \frac{3n}{m} \right) (\epsilon'_x + \mu \epsilon'_y) \right. \\ & \left. - 3n \int_0^{z=t-m} \frac{\epsilon_x + \mu \epsilon_y}{(t - z)^2} dz \right] \end{aligned} \quad (4-15)$$

By interchanging the subscripts "x" and "y" in Eqs. 4-14 and 4-15, the corrections for the biaxial stress in the "y" direction may be obtained. The sign conventions and the meaning of all terms have been given previously.

PREFACE

Problems associated with residual stresses and their determination have been recognized by the Society of Automotive Engineers for several decades. Division 4 (Residual Stresses and Fatigue) of the Iron and Steel Technical Committee published Technical Report 147 entitled, "Evaluation of Methods for Measurement of Residual Stresses," in September, 1957. A later report (TR-182), "Measurement of Stress by X-Ray," describes in detail the use of the x-ray for residual stress determination.

This manual emphasizes mechanical methods of residual stress measurement. It is intended to guide the engineer in selecting an appropriate method, one that will be most expedient and economical, in solving his particular problem.

Chapters 1 and 2 introduce the various methods and give criteria for selection. Chapters 3-7 give detailed descriptions and analyses of the five important techniques of parting out, layer removal, boring and turning, hole drilling, and x-ray diffraction. Each in turn, especially Chapters 3-6, has special sections that clearly show procedures, pertinent equations for calculations, and specific applications to various type geometries.

Particularly useful is the concluding Chapter 8, which treats of the various techniques and devices utilized in measuring strain, curvature, dimensional, and other parameters necessary to solving the problems of residual stress. Machining and layer removal techniques are discussed not only from the viewpoint of actual theory and procedure but also with the objective of eliminating the spurious effects that can be induced by the processes. Special effort has been made to provide a useful list of references for extended study and research.

This manual brings up to date the most recent information available with respect to residual stress measurement, and its completion is the result of the work of many qualified men. The manuscript was prepared by Prof. T. Elsesser in the Department of Theoretical and Applied Mechanics of

the University of Illinois, at the request of Division 4 of the Iron and Steel Technical Committee of the Society of Automotive Engineers. The Mechanical Methods Subcommittee, whose chairman is G. A. Butz of the Aluminum Company of America, had the task of collecting the information included in this manual. This was accomplished by requesting papers from various individuals. The papers and their authors include:

R. F. Brodrick, Lessells and Associates, Inc., "Layer Removal Method of Residual Stress Measurement."

F. R. Holliday, Chrysler Corp., "Sectioning Methods Using Strain Gages."

R. A. Kelsey, Aluminum Company of America, "Hole Drilling Method."

E. S. Machtmen, La Salle Steel Co., "Other Qualitative Methods."

H. R. Neifert, Timken Roller Bearing Co., "Other Qualitative Methods for Measurement of Residual Stresses."

J. E. O'Donnell, La Salle Steel Co., "Slitting Method."

G. Progulski, La Salle Steel Co., "Determination of Residual Stresses by Measurement of Hertzian Hardness."

J. L. Rembowski, General Motors Corp., "A Report on a Theory and Procedure for Residual Stress Analysis."

V. Weiss and G. Sachs, Syracuse University, "Determination of Residual Stresses in Metals."

Together with these authors and other members of the subcommittee -- A. J. Bush, H. L. Chesney, R. A. Huseby, D. E. Martin, J. G. Roberts, and E. O. Stitz -- Chairman Butz and Prof. Elsesser correlated and organized the information provided. The valuable assistance and contributions of this subcommittee have made the manual possible. Appreciation is also expressed for the editorial services of J. F. Millan, J. Morrow, T. Zajac, H. J. Noble, T. Holt, and W. Evans.

Division 4, Residual Stresses and Fatigue
SAE Iron and Steel Technical Committee

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Chapter 1 - INTRODUCTION

1-1. SCOPE AND PURPOSE OF MANUAL

This manual has been assembled as a ready reference source for the engineer who must measure residual stresses. Although the literature has abundant information on the individual methods of residual stress measurement, this manual is unique in that most of the useful methods have been selected, described, classified, and compared.

Methods of measuring residual stress may be classified as mechanical, physical, or chemical. Mechanical methods that are most widely used involve the machining of the part to release the residual stresses. These methods are considered in detail and occupy the bulk of this manual.

Of the physical methods, x-ray diffraction has many advantages in solving some residual stress problems. One of the major advantages is that it is completely nondestructive if only surface residual stresses are desired. This method is briefly described and its limitations discussed so that the investigator may choose this method if it fulfills his needs. The method and the necessary techniques have been covered in detail in Ref. 1.

Of the chemical methods, the stress corrosion cracking technique seems to be the most useful. This cracking is known to be dependent upon the surface tensile stress level. At present it is only a qualitative measure of the stress, but perhaps at some future date, quantitative chemical methods may be developed.

To make residual stress measurements, the engineer must:

1. Select the most appropriate method.
2. Completely familiarize himself with the theory behind the method chosen.
3. Learn the techniques of measurement used by previous investigators.

Note that the word "method" is reserved to mean the individual category of stress analysis, and that "technique" applies to the experimental procedure.

The choice of the method depends to a large extent upon the geometry of the specimen, the precision desired, the type and location of the stress it is desired to measure, and the equipment available. The numerous mechanical methods available are classified into four groups. These are: the parting-out method, the layer removal method, the boring and turning method, and the hole drilling method. The nonmechanical x-ray method is briefly discussed here for comparison.

Once the method has been selected, measurements should not be taken and their values blindly substituted into equations. Each term in the expressions relating the deformation to the residual stress should be completely understood and the sign convention used for these terms should be well known. For this reason, a derivation for each method has been included with the necessary assumptions and limitations. Following a study of these equations, it may be decided whether some automatic or computer technique

should be used. In some instances in which a large number of specimens are to be analyzed, the computer technique offers obvious advantages. At this point, it is also necessary to determine the precision with which the stresses are to be determined. This, of course, can be related directly to the precision used in the measurement of the deformation changes and the original dimensions of the specimen itself.

The correct choice of the many and varied techniques for measuring dimensional changes used in the determination of residual stress is extremely important if accurate and reproducible data are to be obtained. Machining stress introduced in dissecting must not be larger or more significant than the stress that is to be measured. The techniques used to remove material are numerous, and some of the better currently used techniques are considered. The specific techniques associated with any particular method are included and special fixtures and procedures are described and sample calculations are given.

1-2. DESCRIPTION OF RESIDUAL STRESSES AND THEIR EFFECTS

Residual stresses as treated herein will pertain to macrostresses* contained within a body or part. These stresses are of identical nature to the stress produced in a beam or an axial tension or compression member that is subjected to loads. They may be thought of as a statistical average of the stresses across a particular finite length in the material. On the other hand, microstresses or intergranular stresses are the stresses that occur within grains or between grains in a metallic member. These stresses may be thought of as being some superimposed wave or jagged saw-toothed pattern developed over a given length, fluctuating about a mean stress, which is the macroscopic stress. Although microstresses may be important in problems such as fracture, present mechanical methods cannot measure this type of stress.

A residual stress will be further defined as one that occurs because of a local misfit of the material in a single part and not one that is induced in a structure because of assembly misfits.

A residual stress is formed in a member when one portion of the member undergoes a permanent dimensional change. The portion that becomes dimensionally changed is thereafter partially restrained, causing stresses on it and on the portion that has not undergone a size change.

This misfit of one portion of a member relative to the rest of the member may occur when local inelastic deformation occurs, when a member is nitrided or carburized, or when the internal structure of the material itself undergoes a phase change. Some examples of the phenomena that may

*See definition of macrostress and microstress, SAE Quarterly (March, 1963), 118.

induce or alter the residual stress are: coating, forming, drawing, quenching, welding (assembly or repair), straightening, machining, grinding, plating, shot peening, service loads beyond the elastic limit, and localized or general temperature gradients.

Residual stresses can have either a beneficial or a deleterious effect on the performance of a part or structure under load. The beneficial effect of inducing compressive residual stresses in a member that is to be subjected to fatigue loads is well known (2)**. Under static loads, the elastic strength of a member may be considerably increased by inducing residual stresses in the part by a preload in the same direction as the direction in which the major design load is to be supported. Detrimental residual stresses may contribute to brittle fracture, warpage, or stress corrosion. If tensile residual stresses occur at critical areas in a member subjected to fatigue, its life may be greatly reduced. Warping during machining is a serious problem often caused by residual stresses.

This manual should be of considerable assistance in measuring residual stresses to determine the effectiveness of treatments intended to remove unfavorable stresses or in measuring the magnitude and distribution of a favorable system of residual stresses that are intentionally put into the part to increase its load capacity.

1-3. BRIEF DESCRIPTION OF METHODS OF RESIDUAL STRESS MEASUREMENT

1-3.1. THEORY OF DISSECTION METHODS - Mechanical methods used to measure residual stresses necessitate the measurement of deformations that take place as the part is cut or sectioned.

**Numbers in parentheses refer to References at the end of the manual.

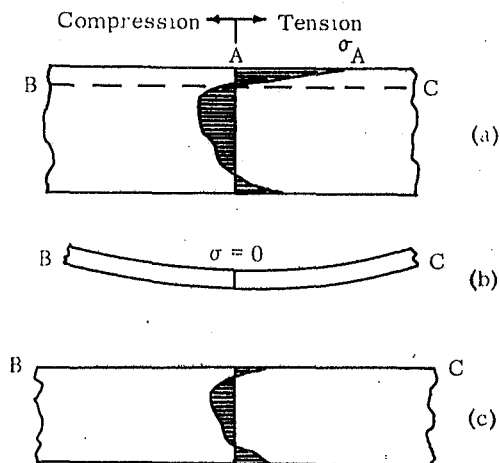


Fig. 1 - Deformation and stress distribution changes due to cut BC

In the direct approach, the stressed layer is removed and the deformation measurements are made on the layer that contained the stresses of interest. In the effect approach, it is necessary to remove, and usually destroy, the material in the layer where the stresses are to be determined; consequently the deformations are measured upon the part remaining, which is herein called the "parent part."

As an example of these two basic approaches, consider the beam in Fig. 1(a), which has the balanced residual stress pattern shown. Let us assume that the residual stress at A is desired. A cut may be made along BC, preserving the thin slab of material in which σ_A occurs. The stress at A is released, and consequently deformation of the slab occurs as shown in Fig. 1(b); this may be measured and the stress at A calculated. The second approach involves removing the thin slab in which σ_A is present and measuring the deformation of the part remaining, as shown in Fig. 1(c). A combination of these two techniques may also be used.

The residual stress σ_A should be thought of as occurring at some point (such as A) on the uniaxial stress-strain curve shown in Fig. 2. If the stress at A is reduced to zero, unloading will occur along a line AB. With a known value of the modulus of elasticity, E, a strain change ($\Delta\epsilon$) will occur on the upper surface of the member. This strain change will be in a direction opposite to the original residual stress.

1-3.2. ASSUMPTIONS OF ALL DISSECTION METHODS - Some of the assumptions necessary in determining the residual stresses, using these methods, are:

1. Linear unloading relation of stress versus strain, with a slope equal to the initial modulus of elasticity, E, as is shown in Fig. 2. This assumption may introduce small errors if a member has undergone considerable inelastic deformation before unloading, owing to the Bauschinger effect shown in Fig. 2.

2. From the microscopic level of observation, it is also necessary to fulfill the conditions of the theory of elasticity in that the material must be homogeneous and isotropic. This condition is not satisfied for a single grain of material, but when a large aggregate of grains are considered, this is a reasonable assumption. A considerable variation in prop-

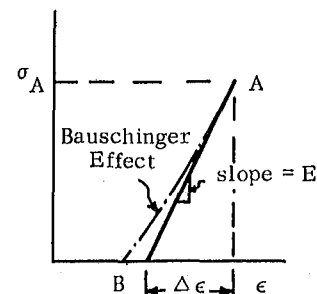


Fig. 2 - σ - ϵ unloading curve for released residual stress σ_A

erties of the material may occur from one point to another as, for example, in the region of a weld in case-hardened steels or in severely cold worked parts; this heterogeneity may introduce errors in the calculations of the residual stresses contained in these parts.

3. In many cases a simple state of uniaxial stress is assumed. If this is done, and biaxial or triaxial stresses actually exist, large inaccuracies can result. The further assumption that the principal stresses remote from the surface have the same directions as principal stresses at the surface may also introduce inaccuracies.

4. In any mechanical method, the significant stresses must act over a large enough region of the material to permit the strains to be measured. The measurement of residual stresses at abrupt changes in sections, such as sharp fillets or keyways is nearly impossible.

5. The stresses induced as a result of sectioning operations do not seriously affect the deformations used to compute the initial residual stresses.

6. Other assumptions necessary in the derivation of each particular mechanical method of residual stress measurement are considered and mentioned, with the derivation of the relation between deformation and residual stress.

This manual is mainly concerned with the measurement of uniaxial or biaxial stresses, and most of the methods described are inadequate to measure the triaxial state of stress below the surface or any deviation in the direction of the principal stresses in the interior of the body. The errors involved are relatively small near the surface.

1-3.3. EQUILIBRIUM AND BOUNDARY CONDITIONS
FOR RESIDUAL STRESSES - To assist in visualizing residual stresses and their effects on the distortion of members, consider a simple member and the limitations imposed on the stresses contained within this member. In general it is necessary to assume that the stress field is uniform or has some degree of uniformity over some length. The member shown in Fig. 3 may be considered to be a plate or section of a prismatic bar. Let the origin of the x , y , and z axes be placed inside the plate at O ; assume that a cubical element exists at O and contains the three principal stresses σ_x , σ_y ,

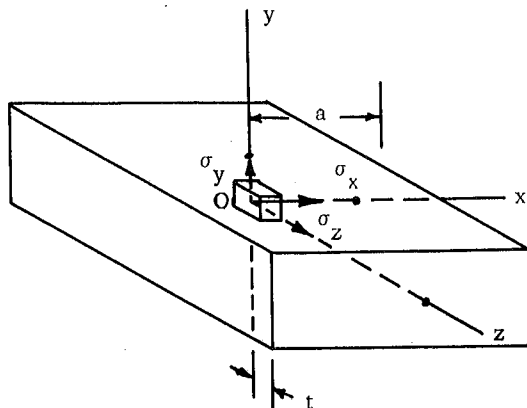


Fig. 3 - Particle of material within solid

and σ_z . Further assume that these principal stresses maintain their initial directions. All stresses are assumed independent of z ; or, in other words, the stresses on the elemental block are constant as the block is translated along any line parallel to z .

If this cubical element is translated along any axis, the corresponding stress vanishes at the boundary; for example, $\sigma_x = 0$ at $x = a$. Also, the stresses on a plane perpendicular to a principal axis must balance or must satisfy the conditions of equilibrium. On planes parallel to the yz plane, the distribution of σ_x stresses may be as shown in Fig. 4.

The conditions of equilibrium are

$$\Sigma F_x = 0$$

$$\Sigma M_z = 0$$

which must be satisfied by this stress distribution. Since no external loads or moments act upon the part (except the negligible loads due to its own weight), the first equation may be rewritten as

$$\int \sigma_x da = 0$$

that is, the algebraic area under the stress distribution curve must be zero. This could also be stated as: The resultant tensile force must equal the resultant compressive force.

The second equation may be written as

$$\int \sigma_x y da = 0$$

which states that the moment of these areas is zero. This may also be stated as: The moment of the resultant tensile force must be equal and opposite to the moment of the resultant compressive force on this section.

Similarly, if the planes are passed through the part parallel to the xz or xy planes, the stresses must "balance" in an exactly similar manner, since no external loads exist. An infinite number of stress distributions will satisfy these equations of equilibrium, but if the stress distribution is par-

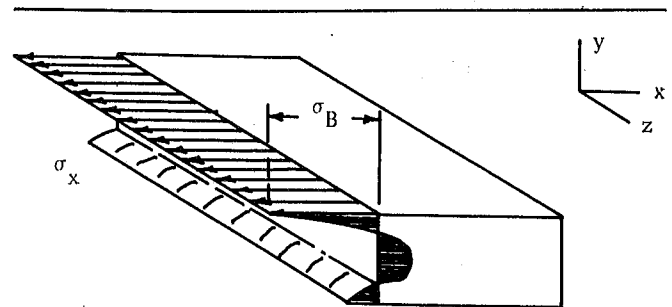


Fig. 4 - Assumed σ_x distribution which is independent of z

tially known or may be considered to be symmetric due to symmetrical treatment (such as quenching or cold working), an intelligent guess of the distribution may be made.

Now consider what happens when a member containing residual stresses is cut by some machining process such as sawing. Using the same logic as before, the normal stresses on the cut surface must necessarily vanish, and the conditions of equilibrium must now apply to the separated pieces.

For the rectangular solid shown in Fig. 3, the stresses were considered to be independent of z ; therefore a cut perpendicular to this direction will not produce any distortion of the plate except in the immediate region of the cut. If the cut is made at a section close to the edge of the plate, σ_z may be considered to be zero through the thickness "t" of the removed strip if "t" is small relative to the depth of the original plate. Neither this strip nor the plate remaining will undergo a curvature change when the σ_z stresses are released, since the summation of these stresses were zero on this plane before cutting.

If a thin sheet is removed from the plate by a cut perpendicular to the y axis, a distortion will occur. Should restraining forces be used to hold in the sheet the stresses that exist before the cut is made, the σ_x and σ_z stresses may be distributed as shown in Fig. 5. Although σ_x is assumed to be constant for all values of z , σ_x will vary with x across the width of the plate. Let it be further assumed that σ_x is reasonably independent of x within the region where the deformation measurements are to be made, say, from AB to A'B' and that the top surface has a state of biaxial tension. The stress in the y direction may be assumed to be zero through the thickness, if the specimen is thin. To satisfy equilibrium in the x direction, σ_x assumes a new pattern that will let the stress distribution reach its minimum state. That is, as the restraint offered by the remainder of the part is released, the distribution changes from the pattern shown in Fig. 6 to that shown in Fig. 7(b). The same type of trans-

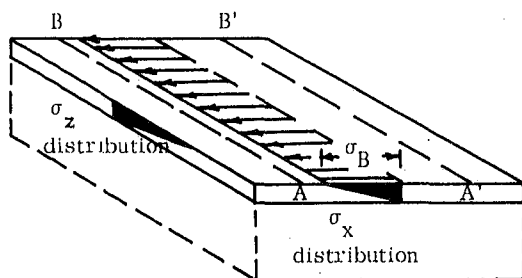


Fig. 5 - Assumed σ_x and σ_z distributions (maintained by restraining forces, not shown) in thin sheet at top of block (Fig. 3)

lation and rotation to satisfy equilibrium will occur in the stresses σ_z .

As these stresses are relieved, the associated strains will cause distortion of the sheet. In the translation, Fig. 7(a), the tensile stresses are released or reduced, which means that the sheet contracts. The rotation of the stress distribution, Fig. 7(b), further reduced this tension and distorts the sheet into a shallow dish.

The previous discussion leads to a relatively important concept, which assists any investigator in the understanding of the deformation that accompanies the sectioning of a specimen. This concept can be stated as follows:

If the residual stress distributions are reversed and assumed to act at the cut faces of a sectioned specimen, these stresses (loads) will produce the distortions observed in the specimen.

To illustrate this concept, the tensile stress distribution shown in Fig. 6 could be reversed (changed to compressive stresses) and assumed to act only at the edges of the sheet parallel to the "z" axis. The same thing could be done with the σ_z stresses. The result would be to distort the sheet into a shallow dish, as described above.

1-3.4. COMPONENTS OF RESIDUAL STRESS - To determine the complete residual stress pattern in a member, it is generally necessary to part out a specimen or coupon to be used in the residual stress analysis (see Chapter 3). During this parting-out operation, strains are usually released in the member. These strains should be measured so that

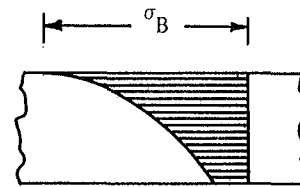


Fig. 6 - σ_x stress distribution in sheet (see Fig. 5)

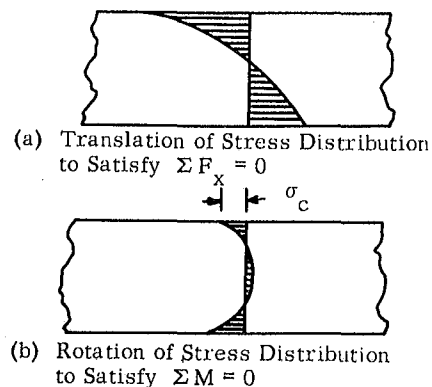


Fig. 7 - Translation and rotation of σ_x distribution

the parting-out component of stress may be evaluated through the thickness of the specimen. This will generally be an assumed linear variation of stress through the thickness of the part, and may be all that is necessary in a given study to evaluate the "average" stresses in the part.

If a more complete stress analysis is desired, the layer component of stress may be measured, using the layer removal method (possibly in conjunction with the x-ray method), the boring and turning method, or the hole drilling method. The total residual stress in the part is obtained by superimposing the parting-out component of stress on the layer component.

1-3.5. CLASSIFICATION OF ESTABLISHED METHODS -

(a) Parting-out Method - This method includes slitting, slotting, ring and tube techniques, and generally requires the removal of a coupon from the parent part in the region of interest. Frequently the stress gradient is assumed to be linear through the thickness of the coupon, and thus the strain change in the coupon will give a good measure of the residual stress. In other cases, the coupon may be considered to be stress free.

(b) Layer Removal Method - The layer removal method is most useful in determining the residual stresses in flat plates or beams in which the stress varies with thickness; particularly if the stress gradient is sharp. This method consists of removing thin layers in the region of interest and observing the dimensional changes in the part from which the layers were removed. In this way the average stress in each layer may be calculated and thus a plot of the residual stress distribution may be obtained.

(c) Boring and Turning Method - This method applies particularly to circular cylindrical sections. As material is removed from the bore or outer cylindrical surface of a shaft, the longitudinal and circumferential deformational changes of the cylinder are determined on the surface opposite to that on which the layers are removed. The tri-axial residual stresses and their distribution may be determined along a radius of the cylindrical member from these deformational measurements.

(d) Hole Drilling Method - In the hole drilling method surface strains in the vicinity of a hole are measured as the hole is drilled. These strains may be used to calculate the biaxial stresses and their distribution at or near the surface of the part. This method permits a survey of the residual stresses over the surface of a large structure. Hole drilling may be used in a qualitative sense by coating the surface of a large part with brittle lacquer and observing the crack formations around the drilled holes. In some cases, a part may still be usable after this analysis.

(e) X-ray Diffraction Method - This method is described in detail elsewhere (TR182) and is included here to permit comparison with the four previously mentioned mechanical methods. X-ray peaks are shifted, depending upon the strain existing within the atomic planes, causing diffraction. The atomic spacing can be measured, from which elastic strain and thence residual stress can be calculated. The method is nondestructive if only surface stresses are desired.

1-3.6. METHODS UNDER DEVELOPMENT - The following methods are included to illustrate that progress is being made in the field of residual stress analysis. These methods are highly qualitative and require further development. One of these methods may someday be completely nondestructive and surpass the usefulness and precision of today's methods.

(a) Ultrasonic Method - By the use of specially cut quartz crystals and relatively complex electronic equipment, shear waves having frequencies of the order of 5 Mc may be produced and their velocity measured in metals. In a shear wave, the wave front moves at high velocities perpendicular to the surface and the particles in motion move parallel to the surface. Stressed metals are birefringent to an ultrasonic shear wave in much the same way transparent materials are birefringent with respect to a polarized light source. A polarized shear wave is resolved into two wave components, which lie in the planes of the principal stresses. These wave components move at different velocities, which are dependent upon the magnitude of the principal stresses.

It has been shown elsewhere (3 and 4) that the stress is linearly dependent upon the velocity difference of the wave components in uniaxially loaded aluminum and mild steel specimens. In this same investigation, inelastic deformation had a negligible effect upon the velocity difference. This makes this technique well suited to the measurement of residual stresses.

Sometime in the future, two collimated beams of ultrasonic waves may be focused upon a given particle of material within a solid, and from suitable measurements, the stress state of that particle may be determined nondestructively.

At the present time, this technique is hindered by the fact that birefringence is not only caused by stress but is also produced as a result of anisotropy. For example, the velocity difference of the component shear waves has been found to be very large in a stress relieved, rolled aluminum bar.

(b) Indentation Method - This method, which is commonly called the Hertzian hardness method, involves the measurement of the load imposed upon a spherical indenter in an amount necessary to cause plastic flow. This load is a measure of the residual surface stresses, since flow will occur when the superimposed values of the residual stress and those produced by the indenter reach certain critical values.

The maximum shear stress flow theory is useful in predicting this critical value, and in addition, predicts that this method is useful in measuring only tensile residual stresses. An investigation by Sines and Carlson (5) found small or negligible changes in hardness in compressive stress fields. Increasing tensile stresses have been found to produce a linear decrease in the Hertzian hardness (5 and 6).

A highly refined experimental technique is required to obtain quantitative results, since generally only small changes in hardness are produced by relatively large changes in stress. The hardness value is dependent upon such vari-

ables as:

1. The technique used to establish the onset of flow.
2. A definite value for the flow stress (may be impossible to obtain in certain soft materials).
3. Normal variations in the hardness within a part.
4. Surface finish in the test zone.
5. Part thickness, especially when the part is thin.

Although the method is incapable of distinguishing between uniaxial or biaxial stress states and cannot be used to establish principal stress directions, it is capable of measuring the "average" stress over some small depth near the surface. Due to the simplicity and nondestructive character of a hardness test, development of the method should be encouraged.

(c) Stress Corrosion Method - Corrodents, which cause cracking of the surface of certain metals when a tensile stress field exists, may be used to detect residual stresses. For carefully controlled conditions, the method may be somewhat quantitative, since the time to cracking is roughly proportional to the level of tensile stress. This tensile stress must exceed a certain threshold value; otherwise, the cracking does not occur. Examples of metals and severe corrosive agents are listed in Table 1.

Table 1 - Corrosive Agents on Metals

Metal	Agent	Reference
Brass	Ammonia, air and water vapor,	(7)
	or mercurous nitrate and	(8)
	nitric acid solution	(9)
Aluminum	Mercuric chloride and hydrochloric acid solution	(10)
Stainless Steel	Copper sulfate and sulfuric acid solution	(11) Hard steels (12) Stainless steel (13) steel

The method is highly qualitative, since small changes in composition of the metal or corrodent chemistry have a large effect. In addition, temperature, stress state, surface condition, and metal anisotropy have a large influence on the time required to produce cracking. Although some investigators have obtained reproducible results, further study is obviously needed. The simplicity of the method and of the equipment required would, however, recommend its further development.

Chapter 2 - CHOICE OF METHOD

The nature of the problem involving residual stresses determines, in part, the appropriate method to use. For example, premature fractures in members subjected to fatigue loads may require a precise determination of the surface residual stresses that are developed. However, warpage during processing may require only the determination of some average residual stress across the section, in which case a much less precise method can be used. Stress corrosion cracking may often be solved simply by insuring that the surface stresses are compressive.

Here are some typical questions that should be asked before a method is chosen:

1. Am I primarily concerned with the magnitude of the surface stress, or is it necessary only to know if it is a tensile or compressive state of stress?
2. Am I concerned with the stress at the surface or perhaps the peak stress, which may occur at some subsurface level, or possibly is it the magnitude of the shear stress that occurs at some indeterminate point in the structure?
3. In any case, with what precision must this stress be known to solve the particular problem?
4. Is it close enough, for instance, to have a measurement of stress that is within 50% of the actual value?

Before these questions can be answered intelligently, the investigator must have some concept of the importance of the magnitude, type, state, gradient, and location of the significant residual stress or stresses for his particular problem. A few examples will be discussed below.

Fatigue behavior and its relation to residual stresses has been studied more thoroughly than any other problem. If the residual stress changes gradually in its variation with depth, the residual stress on the surface will be beneficial if it is compressive, and detrimental if it is tensile. If, however, the stress gradient is large at the surface, passing from compression at the surface into tension at a region a few thousandths of an inch below the surface, a subsurface crack may develop in the tensile zone. The maximum compressive stress near the surface should still be considered the

critical stress, since the crack must propagate through this compressive field to get to the surface(2).

If the reversed condition occurs, that is, if compressive stresses occur at a subsurface layer, with tensile stress on the surface, this tensile stress will produce the maximum detrimental effect, greatly reducing the fatigue life of the particular part. Thus it is most important in fatigue problems to know accurately the magnitude of the maximum residual stress near the surface.

The brittle fracture problem is extremely complex, since the flaw size in the member is as important as the state of stress. Most residual stress analyses assume a uniform stress field, which, of course, will not exist in the region of a flaw, and thus the use of most methods to determine a critical or significant stress as related to brittle fracture is difficult. Fracture is often influenced by the microscopic element of the stress, which cannot be measured by the mechanical methods, indicating that the x-ray method be used, since the localized strains in a critical section could be more easily measured.

Stress corrosion is closely related to the surface residual stresses in the part. A survey method is most useful, such as the hole drilling method or perhaps sectioning, using the parting-out method over the entire part. Auxiliary experiments are valuable at this point to determine what degree of stress may be withstood without causing stress corrosion in the material. Actually to determine the stress over the entire surface of a complex part is a tedious and time-consuming operation. However, a qualitative approach may be used, making it possible to approximate the conditions necessary for stress corrosion to occur.

Warpage is another important problem and may occur as a part is machined or as a straightened member is heat treated. Some "average" residual stress, which may be measured by the parting-out method, may be closely correlated to the warpage.

After deciding what stress is significant and the precision needed in its determination, Table 2 should be of further assistance in choosing the method to be used. Obvious practical considerations such as geometry of part and equipment needed for the analysis are also included in Table 2.

Table 2 - Comparison of Methods

	Parting Out ^{1, 2b} (including Slitting, Slotting, Tube and Ring Methods)		Layer Removal ^{2a}		Boring and Turning ^{2a}	Hole Drilling ^{2a}	X-Ray ^{2c, 8}
Shape of Parent Part in Region of Interest	Free from abrupt section changes or treatments		Free from abrupt section changes		Circular cylinder	Nearly flat surface	None, except limited by geom- etry of setup
Shape and Orientation of Coupon in Parent Part	Beam ⁵ oriented in direction of principal stress or in direction of interest. May be straight or curved Any const. cross- section if all stresses are as- sumed cons't; otherwise must be rectangular	Plate ⁶ or Shell flat, slightly curved, or cy- lindrical, Thickness small and constant	Beam ⁵ oriented in direction of prin- cipal stress or in direction of in- terest. May be straight or curved. Any const. cross- section if all stresses are as- sumed cons't; otherwise must be rectangular	Plate ⁶ or Shell flat, slightly curved, or cy- lindrical. Thickness small and constant	Circular cylinder length > 2 × dia.	No coupon necessary	No coupon necessary
Usual Minimum Size of Coupon	1 in. long	1 in. square	1 in. long	1 in. square	1 in. OD	Constant at surface and low stress gradient	Constant in x-ray beam (~ ¹ / ₁₆ in. dia) and 0.0005 below surface for steel
Stress Field Assumed (3)	Vary in direction of thickness only, and vary linearly or have known nonlinear distributions		Vary in direction of thickness only		Radially symmet. and constant with length	Average surface stresses or stress distribution to small depth	Surface stresses unless part is dis- sected ⁷
Stresses Measured in Coupon (4)	Surface or average longitudinal stress in thickness	Surface or Average Stresses in Thickness	Longitudinal stress versus thickness	Stresses having directions parallel to sur- face versus thickness	Stresses in three principal di- rections versus radius		
Conversion of Coupon Stresses to Biaxial stresses in Part (not req'd if stresses are uniaxial)	Two beams needed parted-out in perpendicular direction unless degree of biaxial- ity is known	None	Same as parting- out beam	None	None	None	None
Ideally Suited to	Survey of average stresses in large part		Precise measurement of surface stresses and stress gradients		Triaxial stresses versus radius	Survey average stresses at sur- face of large part	Same as layer removal method

Precision of Stress Measurement	Error may be very large if stress gradient is steep and changing. "Average" stresses usually determined however in some cases may be quite accurate.	Error small. Precision improved by using primary and secondary layer removal when steep stress gradients exist.	Error reduced by combined boring and turning but stiffness of tube reduces accuracy when steep stress gradients occur.	Error large if stress gradient is steep at surface. Best for average surface stress	Error small with correct technique see Ref. 1
Deformational Measurements	Curvatures only if coupon contains full thickness of part; otherwise curvature and strain changes are needed. In flat plate, strain rosette is needed if principal directions are unknown. Measurements are to be made at locations spaced in from the edge (plate) or ends (beams) a distance at least equal to thickness.	Curvature changes or strain changes versus thickness removed. Measure in from edge (plate) or ends (beam) a distance equal to thickness.	Long, and circ. strains-measure at mid-length of cylinder	Strain rosette at hole unless principal directions are known.	2, 3, or 4 "exposures" at different x-ray beam angles. Only elastic component of strain is measured
Complexity of Analysis	Simple	Relatively complex, especially for thick curved beam. May warrant computer programming if several specimens are to be studied.		Medium	Medium
Equipment Needed	Special cut off wheels for horizontal grinding -- strain gages -- curvature gages. Machine shop -- hack saw, jeweler's saw	Grinding, etching or electropolishing facilities, tooling, and measuring lab. -- air conditioned for best results. Precision curvature gage and strain gages. Machine shop		Strain gages End mill or drill	x-ray diffractometer Cr-K α and V-filter for steel sample

¹ This method may be required in conjunction with any of the other methods to obtain a complete determination of the residual stresses in the parent part; or, on occasion, if the stress pattern in the part is known or can be reasonably assumed (in both direction and distribution versus thickness) a reasonably accurate determination of the residual stress may be obtained using this method alone.

² All methods are based upon the theory that the stresses in an increment of thickness can be measured by deformations produced and measured on the:

- (a) Parent part (or a coupon removed from the parent part) by the removal of thin stressed layers or by drilling holes.
- (b) Layer itself by the removal of the parent part (thus removing the restraint produced by the parent part).
- (c) Layer itself by measuring atomic lattice parameters.

All methods are destructive if stress versus thickness is required: while x-ray may not be if only surface stresses are desired. Hole drilling may be classified as semidestructive.

³ All methods neglect or are inadequate to measure the principal stress whose direction is perpendicular to the surface, with the exception of the boring and turning method.

⁴ A rough surface makes the exact depth of a surface stress questionable. Smoothing or cleaning by grinding or lapping, though desirable, may either remove the sought after stress or introduce significant extraneous stresses. Electropolishing is best for x-ray.

⁵ Beam coupons have a length which is at least 5 times its maximum lateral dimension.

⁶ Plate coupons have edge dimensions which are at least 10 times its thickness.

⁷ Microstress component of stress in multiphase materials may hinder measurement of macrostress component.

⁸ X-ray method is actually a strain measuring technique and as such may be used in conjunction with any other method.

3-1. DESCRIPTION

The parting out method is used as:

1. The first step in a detailed residual stress determination, using one of the methods discussed later.
2. A method complete in itself to measure residual stresses.

Consider item 1 for a moment. In some members, parting-out will not release significant residual stresses. For example, to determine the stresses in a round cylindrical shaft by using the boring and turning method, it is necessary to use the parting-out method to remove a short length of this shaft. Usually in this case, deformation measurements are unnecessary during parting-out and the parting-out component of stress is zero.

On the other hand, if a thick slab is removed by sawing from the surface of a large casting, the deformation released by this cutting-out operation must be used to evaluate the parting-out component of stress. If the plate is to be analyzed subsequently by the layer removal method, the stresses in the casting are found by superimposing the parting-out components and the layer components of residual stress.

As a general rule, it is necessary to obtain deformation measurements during parting-out if either loads or moments are released. Other considerations preliminary to parting-out are also given in this Chapter. The parting-out component of stress may be determined by using the methods described in the succeeding paragraphs.

As mentioned in item 2, the parting-out method may sometimes be used alone to measure residual stresses. The method requires removal from the parent part of a coupon that has the shape of a flat plate or a straight or curved beam. Residual stresses in the parent part may be determined if the distribution of the stresses through the thickness of the coupon is known or can be reasonably assumed.

In addition to the assumptions given in Chapter 1 of this manual, it is usually assumed that the stresses are constant in planes parallel to the surface of the part and that the stress distributions on planes perpendicular to the surface are linear. Section 3-5 presents a technique that may be used when the stresses have a known curvilinear distribution.

The type of coupon to be parted-out depends on the state of stress to be measured and on other factors such as parent part geometry. Plate or beam coupons may be used to measure either uniaxial or biaxial residual stresses. Plates must be used if the residual stresses have completely unknown principal directions. Only one beam is needed when the stresses are uniaxial, while two beams are generally needed if the stresses are biaxial. The succeeding sections will pertain to both plate and beam coupons.

3-2. GENERAL THEORY FOR PLATE AND BEAM COUPONS.

Fig. 8 shows a reasonably flat plate coupon that was removed from a large block of material in which the residual

stresses are to be determined. It is desired to evaluate the linear distribution of the residual stresses in the two principal directions through the entire thickness of the coupon. Since the principal stress and strain directions are coincident, these directions may be obtained by the use of strain rosette or curvature measurements. The procedure for principal stress determination is given in standard experimental stress analysis texts, and it will be assumed at this point that the principal stress and strain directions are known.

Let "x" and "y" in Fig. 8 be the principal directions on the surface of the coupon. As illustrated in Figs. 6 and 7(a), the plate will shorten or lengthen in these principal directions, depending upon the average stresses across the thickness of the coupon. As further illustrated in Fig. 7(b), the rotation of the stress distribution will tend to produce "bending" in the "x" or "y" directions, depending in part upon the difference between the stress on the inner and outer surface of the plate coupon. If a linear distribution of the stresses exists in Fig. 6, the stresses will completely vanish after parting-out has been accomplished.

For this case, the strains released at the surface will be a true measure of the biaxial surface stresses. The principal stresses may be determined from the equations:

$$\sigma_x = \frac{E}{1 - \mu^2} (\epsilon_x + \mu \epsilon_y) \quad (3-1)$$

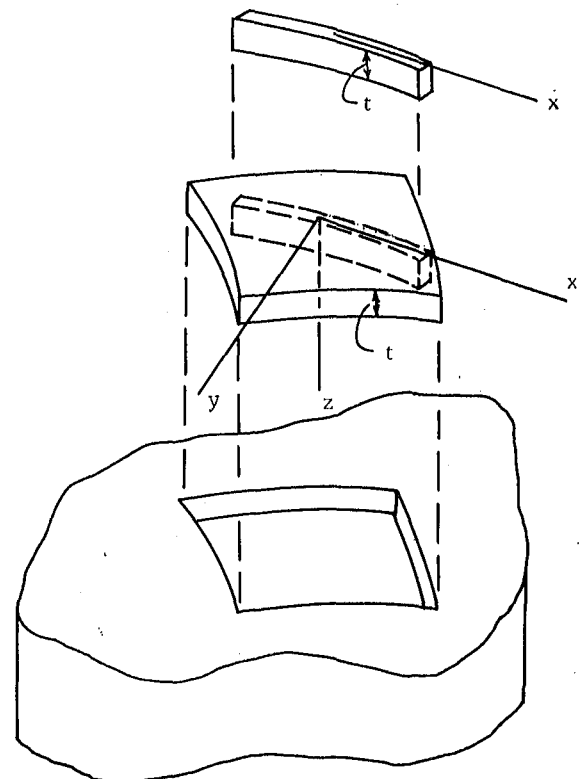


Fig. 8 - Parting out of plate or beam specimen

$$\sigma_y = \frac{E}{1 - \mu^2} (\epsilon_y + \mu \epsilon_x) \quad (3-2)$$

where:

E = Modulus of elasticity

μ = Poisson's ratio

The principal surface strains, ϵ_x and ϵ_y , are positive if the strain readings decrease, indicating that tensile residual strains exist.

To obtain the stress distribution in the principal directions through the thickness of this plate coupon, it is necessary to evaluate the principal strains released in the "x" and "y" directions as a function of the depth "z" below the surface. Since it is generally impossible to measure the strains on the bottom face of the plate coupon, curvature measurements are necessary in the principal directions.

The curvature change produced by parting-out is a measure of the strain gradient, which is the slope of the elastic strain distribution in the part before parting-out (Section 3-4). In addition, since over a given chord length the arc height rather than the curvature is usually measured, equations are also given in Section 3-4 which relate curvature and arc height. As illustrated in Fig. 9, the strain at any depth "z" below the surface may be expressed in the principal "x" direction as

$$\epsilon_x(z) = \epsilon_x - C_x z$$

A similar expression exists in the "y" direction and is

$$\epsilon_y(z) = \epsilon_y - C_y z$$

The curvatures C_x and C_y are positive when the center of curvature is above the surface and decreases during the parting-out. For example, the curvature change is positive if a flat plate becomes convex downward as a result of the parting operation. If a small initial curvature exists, C_x and C_y are the change in curvature, with convex downward values positive.

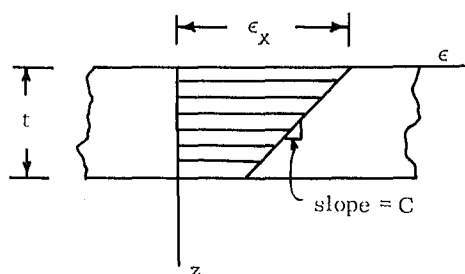


Fig. 9 - Linear strain distribution released by parting out

Using the biaxial stress-strain relations given by Eqs. 3-1 and 3-2, the linear stress distributions in the principal directions may be expressed as

$$\sigma_x(z) = \frac{E}{1 - \mu^2} \left[\epsilon_x - C_x z + \mu(\epsilon_y - C_y z) \right] \quad (3-3)$$

$$\sigma_y(z) = \frac{E}{1 - \mu^2} \left[\epsilon_y - C_y z + \mu(\epsilon_x - C_x z) \right] \quad (3-4)$$

These equations may be greatly simplified if the stress state is assumed to be uniaxial. Assuming that only $\sigma_x(z)$ exists, then $\epsilon_y = -\mu \epsilon_x$ and $C_y = -\mu C_x$; So, Eq. 3-3 simplifies to

$$\sigma_x(z) = E(\epsilon_x - C_x z) \quad (3-5)$$

This equation may be used in conjunction with a plate specimen or a beam specimen oriented in the "x" direction, as shown in Fig. 8.

In some special problems, discussed later, equilibrium of the stress distributions requires that a zero strain change occur at $z = t/2$. In this case,

$$\epsilon_x - C_x t/2 = 0$$

and

$$\epsilon_y - C_y t/2 = 0$$

Solving these equations for the surface strains (ϵ_x and ϵ_y) and substituting in Eqs. 3-3 and 3-4, we obtain

$$\sigma_x(z) = \frac{E(t/2 - z)}{1 - \mu^2} \left[C_x + \mu C_y \right] \quad (3-6)$$

and

$$\sigma_y(z) = \frac{E(t/2 - z)}{1 - \mu^2} \left[C_y + \mu C_x \right] \quad (3-7)$$

For this special case, if we further assume that uniaxial stresses exist, then $C_y = -\mu C_x$ and Eq. 3-6 simplifies to

$$\sigma_x(z) = E(t/2 - z) C_x \quad (3-8)$$

To assure the correctness of the strain and curvature measurements, the conditions imposed by Saint-Venant require that the strain and curvature measurements be made from the edge of the plate coupon at a distance that is at least equal to the thickness of the coupon. In beam specimens these measurements should be made from the end at a distance that is at least equal to the depth of the beam.

Examples that illustrate the parting-out method will now be presented:

Example 1 - In certain members such as large rolled sections, it may be assumed that only longitudinal stresses exist, and they are reasonably constant in the volume of a cut-out coupon. To approximate the longitudinal residual stress distribution in a rail such as shown in Fig. 10, a number of longitudinal strips are cut. From careful measurements of the length of these strips, the strain distribution is obtained.

Longitudinal stresses may be calculated by using $\sigma = E\epsilon$. The strips should be prismatic, but may have any constant cross-sectional geometry.

Example 2 - As illustrated in Fig. 11, the equilibrium of the stresses in a symmetrically treated bar or ring requires that the stresses released be essentially "bending" stresses. The resultant force T due to the tensile stresses must equal the resultant force C due to the compressive stresses; T and C are the forces of a couple that causes pure bending in these members.

If the ring is cut along a radial line or the bar is split

by a longitudinal cut at the centerline, the release of the stresses across the sections will produce only a change in curvature of the member. The change of curvature of the circular ring may be easily determined from diameter measurements made before and after cutting.

The surface stress may be determined by using a strain gage on the surface, and Eq. 3-5, or the stress may be related to the change of curvature by Eq. 3-8, where "t" is the thickness of the ring or half the thickness of the rectangular bar.

Section 3-6.2 describes the technique used on bars and rings.

Example 3 - In parts in which a uniaxial stress state may be assumed, such as the ring and bar previously described, more precise values of stress may be obtained by using thinner coupons. (See beam B shown in Fig. 11.)

Strain measurements obtained in a longitudinal direction on these coupons, using the technique described in Section 3-6.3, permit evaluation of the uniaxial stress, using Eq. 3-5. Curvature measurements are unnecessary unless the distribution of the stress through the coupon thickness is desired.

Example 4 - A thin walled tube is similar to the ring described above. The tube, however, may have biaxial stresses instead of uniaxial stresses, as assumed in Example 3. Equilibrium requires that the circumferential stresses in a tube be balanced. In addition, the longitudinal stresses, if assumed radially symmetrical, are often of a "bending" nature.

By making a longitudinal cut in a tube whose length is at least twice its diameter, the change in curvature of the tube enables the calculation of the circumferential residual

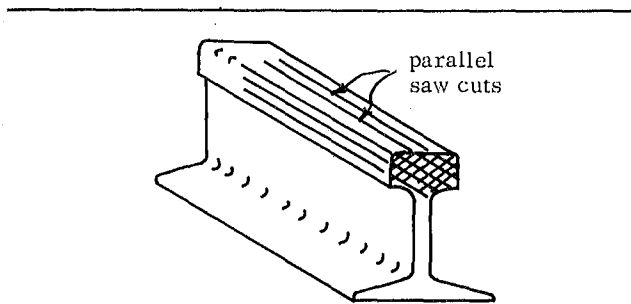


Fig. 10 - Longitudinal saw cuts in rail head to determine longitudinal residual stress distribution

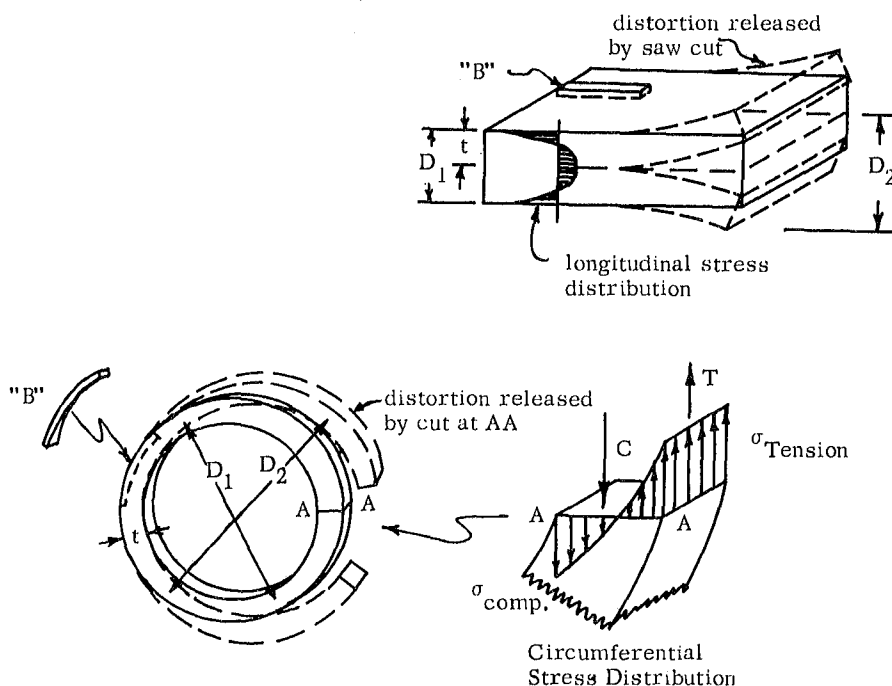


Fig. 11 - Ring and bar in which cuts produce pure bending