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Translation

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Method for determining the equivalent conicity

Méthode de détermination de la conicité équivalente
Methode zur Bestimmung der äquivalenten Konizität



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Summary

In order to be able to compare the results obtained by different railways, both as to the value of the equivalent conicity and as to the test results where this parameter plays an important role, it is necessary for the equivalent conicity to be calculated according to the same principles.

This leaflet sets down the principles of calculation that must be followed, but it does not impose any particular numerical calculation method.

It states the tolerances within which the measurements should generally be made, and beyond that it provides reference profiles and a range of acceptability within which the results of the conicity calculation must fit using the program and taking into account the maximum random errors introduced by the measuring system.

1 - Purpose

The wheel-rail interface is fundamental to explain the dynamic running behaviour of a railway vehicle. It must therefore be understood and among the parameters by which it is characterised, the one called "equivalent conicity" plays an essential role since it allows optimal appreciation of wheel-rail contact on tangent track and on large-radius curves.

As the term equivalent conicity is used in a number of standard documents, including UIC leaflets, ISO standards and European standards, there needs to be an unambiguous way of determining it.

The purpose of this leaflet is to propose a method for determining equivalent conicity which can be applied in the context of standards documents.

This leaflet does not define limits for the equivalent conicity and gives no tolerances for the shape of the railhead and the wheel profile to obtain acceptable results with the conicity.

2 - Symbols, principles and definition

2.1 - Symbols

x	displacement of wheelset in the longitudinal direction of the track
y	displacement of wheelset in the lateral direction of the track
Ψ	angle of movement in the x-y-plane
e	distance between contact points (approximately 1 500 mm for standard gauge)
λ	wave length
r_0	radius of wheels when the wheelset is centred on the track
r_1	rolling radius of right-hand wheel
r_2	rolling radius of left-hand wheel
r	mean rolling radius of both wheels
Δr	difference of rolling-radii between right-hand and left-hand wheels
R	local radius of wheel path
ds	curve length of path corresponding to angle $d\Psi$
$\tan\gamma_e$	equivalent conicity
$\tan\gamma_a$	inclination of wheel and rail profiles in contact point
$\hat{y}$	amplitude of wave
y_{em}	y-displacement, where $\Delta r = 0$
y_{emin}, y_{emax}	corresponding y_{min} and y_{max} displacements for a certain wave
V	speed of forward movement of vehicle

2.2 - Principles and definition

The kinematic movement of a free wheelset, with no inertia, running on a track, is described by the following differential equation:

$$\ddot{y} + \frac{V^2}{er_0} \Delta r = 0$$

Without limiting the conclusions concerning calculation of the equivalent conicity, the speed of forward movement V of the vehicle can be assumed to be constant for the purposes of this study, such that:

$$V = \frac{dx}{dt}$$

hence: $\frac{dy}{dt} = V \frac{dy}{dx}$ and $\frac{d^2y}{dt^2} = V^2 \frac{d^2y}{dx^2}$

The differential equation becomes:

$$\frac{d^2y}{dx^2} + \frac{\Delta r}{er_0} = 0$$

In the case of a wheelset whose wheels have a conical profile of angle γ :

$$\Delta r = 2y \tan \gamma$$

The differential equation then becomes:

$$\frac{d^2y}{dx^2} + \frac{2 \tan \gamma}{er_0} y = 0$$

a second-order differential equation with constant coefficients whose solution is a sinewave with a wavelenght of λ :

$$\lambda = 2\pi \sqrt{\frac{er_0}{2 \tan \gamma}} \quad (\text{KLINGEL's formula})$$

When the wheels do not have a conical profile, linearisation methods can be used so that the linear differential equation can still be applied by replacing $\tan \gamma$ with $\tan \gamma_e$, which is called the "equivalent conicity".

By definition, the equivalent conicity is equal to the tangent $\tan \gamma_e$ of the cone angle of a wheelset with coned wheels whose lateral movement has the same kinematic wavelenght as the given wheelset (but only on tangent track and on large-radius curves).

3 - Method of determination

3.1 - General principles

The following assumptions are used:

- both the wheel and the rail are considered rigid,
- a theoretical wheel is symmetrical in revolution,
- a theoretical rail is straight and is represented by a single profile,
- a real rail is defined by at least 11 profiles regularly spaced apart over a 100 m section of line; the conicity is obtained by taking the average of these individual conicities, the standard deviation of which should also be indicated,
- the wheel does not penetrate into the rail: only point contacts are considered,
- no account is taken of an axle's roll (rotation about an axis longitudinal to the track) as the wheelset moves laterally on the track,
- at the point of contact, the planes tangent to the rail and to the wheel are parallel.

NB : a real wheel can be defined using an average profile determined by taking the average of 4 sections of the wheel located 90 degrees apart.

For any given wheel and rail profile (theoretical or real), it is possible to determine the equivalent conicity $\tan \gamma_e$ associated with the lateral movement $\hat{y}$ of the wheelset on the track.

This is done by the following procedure:

1. determine the wheel and rail profiles, either by measurement for real profiles or by a theoretical calculation for theoretical profiles;
2. determine the $\Delta r = f(y)$ characteristic giving, for each lateral movement y of the wheelset on the track, the difference between the right-hand and the left-hand rolling radii $\Delta r = r_1 - r_2$;
3. determine the equivalent conicity for a lateral movement $\hat{y}$ of the wheelset on the track.

The three above-defined steps of the procedure are explained below.

3.2 - Determining the wheel and rail profiles

The two rails of the railway track and the two wheels of the wheelset are referred to a track-centred coordinate system oriented such that the x-axis is longitudinal, the y-axis transversal and the z-axis vertical.

The wheel and rail profiles must be characterised in a Cartesian coordinate system $((y_i, z_i))$ such that:

- for the rail, the profile is defined not only on top but also on the inner side;
- for the wheel, the profile is defined not only on the classic wheel tread but also on the outer part and in the area of the wheel flange root.

When the profiles are determined by measurement, special-purpose devices can be used, such as wheel and rail profile measuring rules or automatic rail measuring systems carried aboard special railbound vehicles.

It must be stated in the report whether the profiles were measured in the loaded or unloaded condition.

Measurement accuracies must lie within the following tolerances:

- wheel radius : 0,1 mm
- wheel back-to-back distance : 0,5 mm
- track gauge : 0,5 mm
- measured data : they must be consistent with the calculation method (see point 4 - page 9)
- (wheel and rail profiles)

3.3 - Determining the Δr rolling radius difference

To determine the Cartesian coordinates of the wheels, it is necessary to take into account the actual radius of each wheel.

The Cartesian coordinates of the two wheels and the two rails, referred to a track-centred coordinate system, may be computer-processed (smoothing or interpolation in particular) for easier utilisation in the next step of the procedure.

Then, the $\Delta r = f(y)$ characteristic which yields the rolling radius difference between the right and left side for a lateral movement y of the wheelset on the track, is worked out relative to a centred position and for lateral movements up to a maximum corresponding to $\Delta r = r_1 - r_2 = 5 \text{ mm}$, in maximum steps of $\Delta y = 0,2 \text{ mm}$.

The $\Delta r = f(y)$ characteristic is presented as in the example shown in Appendix A - page 12.

3.4 - Determining the equivalent conicity

The equivalent conicity is determined by two-step computation as follows:

3.4.1 - Integration of the differential equation

$$\frac{d^2 y}{dx^2} + \frac{\Delta r}{er_0} = 0$$

using the previously determined $\Delta r = f(y)$ characteristic together with the following initial conditions:

$$y = y_0 \quad \text{for } x = 0$$

$$\frac{dy}{dx} = 0 \quad \text{for } x = 0$$

This integration, based on initial amplitude y_0 , leads to a periodic movement of the wheelset with a peak-to-peak amplitude of $2\hat{y}$ and a wavelength λ .

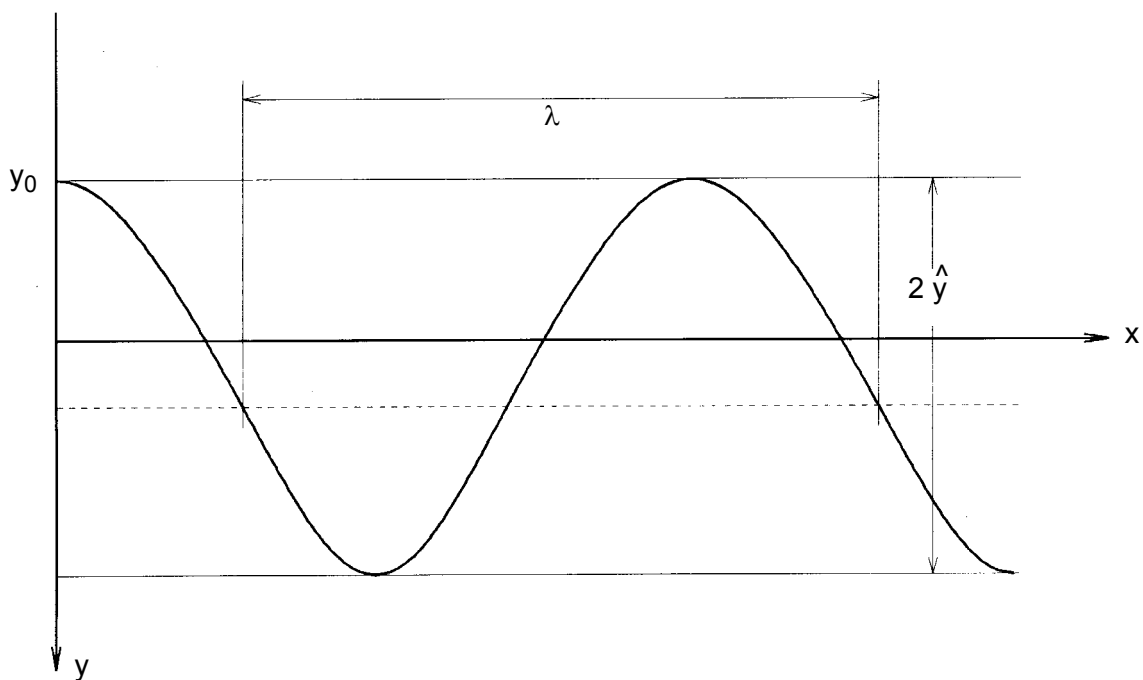


Fig. 1 - $y = f(x)$ function

3.4.2 - Calculation of the conicity

From the actual movement of the wheelset the conicity is computed for amplitude $\hat{y}$:

- either by applying the Klingel formula:

$$\tan \gamma_e = \left(\frac{\pi}{\lambda} \right)^2 2e r_0$$

- or by applying a least-squares type linear regression to the portion of the $\Delta r = f(y)$ characteristic within the $2\hat{y}$ interval.

The slope of this regression is equal to $2 \tan \gamma_e$.

Out of practicality, the conicity is evaluated for changes in $\hat{y}$ from 1 to 8 mm. An example of graphic representation of this exercise is also given in Appendix [A - page 12](#).

The maximum value 8 mm may never be reached if the $\Delta r = f(y)$ characteristic is either highly nonlinear or exhibits substantial jumps.

In Appendix [B - page 13](#), an example is given for a method which is based on determining the wavelength and calculating the equivalent conicity by the KLINGEL formula.

In Appendix [C - page 21](#), hints are given for a method which is based on a linear regression of the Δr function.

In this leaflet no procedure is dictated for calculating the equivalent conicity. But the user of any procedure must be sure that the applied method leads to an equivalent conicity in phase with the definition which is given in this leaflet ([see point 2 - page 3](#)), hence the necessity of reference profiles and results as presented in [point 4 - page 9](#).

4 - Benchmark calculation

For benchmark purposes, reference profiles are defined. In Appendix D - page 23, mathematical formulas are given for wheel profiles A, B, H and I and for rail profile A in the same way as the drawing and the Cartesian coordinates for each profile.

NB : the reference profiles given in Appendix D are only defined for benchmark calculations to test the software in a wide range of conditions. They have no practical relevance for the design of a real profile.

In Appendix E - page 33, the calculated results of the Δr function and the equivalent conicity, according to Klingel's formula, are given for the following wheel/rail-combinations:

1 - wheel A / rail A	
2 - wheel B / rail A	
3 - wheel H / rail A	
4 - wheel I / rail A	
5 - wheel A / rail A	difference of left wheel-diameter 2 mm
6 - wheel B / rail A	difference of left wheel-diameter 2 mm
7 - wheel H / rail A	difference of left wheel-diameter 2 mm
8 - wheel I / rail A	difference of left wheel-diameter 2 mm
9 - right wheel A, left wheel B / rail A	(as an example for an asymmetrical Δr function)

For each wheel/rail combination, the following drawings are given in points:

Appendix E.x.1 (points E.1.1, E.2.1, E.3.1, E.4.1, E.5.1, E.6.1, E.7.1, E.8.1, E.9.1):

$\Delta r = f(y)$	difference of the roll-radii, necessary for calculating the equivalent conicity
$\tan \gamma_a = f(y)$	contact angle for the right and the left profiles as well as the sum of the contact angles, not necessary for calculating the equivalent conicity, but important for independent wheels (as an example)
$\tan \gamma_e = f(y)$	equivalent conicity
representation of the contact points.	

Appendix E.x.2 (points E.1.2, E.2.2, E.3.2, E.4.2, E.5.2, E.6.2, E.7.2, E.8.2, E.9.2): representation of the roll-curves (normally not necessary).

Appendix E.x.3 (points E.1.3, E.2.3, E.3.3, E.4.3, E.5.3, E.6.3, E.7.3, E.8.3, E.9.3) gives the numerical values for the function $\Delta r = f(y)$.

Appendix E.x.4 (points E.1.4, E.2.4, E.3.4, E.4.4, E.5.4, E.6.4, E.7.4, E.8.4, E.9.4) gives the numerical values for the function $\tan \gamma_e = f(y)$.

In this UIC leaflet no procedure is defined for calculating the wheel/rail contact geometry, but the accuracy of the calculation method adopted must be tested.

Because of the accuracy of the measuring equipment, a smoothing procedure must be included in the calculation method or in the pre-calculation stage. The smoothing procedure depends on the measuring system adopted.

The tolerances for the calculation method, including the effects of the smoothing procedure, are given in Appendix F - page 69.

The tolerances are based on the following formula:

$$\Delta(\tan\gamma_e) = \pm \min(0,05 ; 0,5 \tan\gamma_e)$$

In of a steep slope of the $\tan\gamma_e$ function, these tolerances are very difficult to fulfil. In this case, for the benchmark calculation only, additional tolerances of Δy on the y deviation may be considered. Δy is calculated by the following formula:

$$\Delta y = a_0 \left(1 - \cos \left(\arctg a_1 \frac{d \tan \gamma_e}{dy} \right) \right)$$

where: $a_0 = 0,2 \text{ mm}$ and $a_1 = 10 \text{ mm}$.

The tolerances for a certain y amplitude as calculated by the formula $\Delta(\tan\gamma_e)$ must be increased by a value $\pm\Delta y$, keeping the maximum tolerance in this range.

If the situation of a steep slope occurs when applying *UIC Leaflet 518* (see Bibliography - page 94) at the $y = 3 \text{ mm}$ point, then further measurements and/or calculations must be undertaken to understand the practical condition.

The validation of the smoothing procedure is related to the measuring system (for example resolution and measuring directions) and must be defined on the basis of parameters of the measuring devices, but the influence of a random error and the influence of a grid (which is given by the resolution of the digitizing) must be included in the validation procedure.

In both cases, the degree of error should be modulated to determine the necessary accuracy of the measuring devices. This operation involves combining the smoothing and calculating procedure acceptable results referring to the given tolerances.

The calculations shall be made for all reference-profile combinations as given in Appendix E - page 33, and include the influence of asymmetrical Δr -functions produced by different wheel diameters or different wheel profiles on both wheels as given in points E.5 - page 49 to E.9 - page 65.

The effect of a random error and a grid shall be proven by separate calculations for the cases where an error occur only at the wheel, only at the rail and both at wheel and rail.

The effects shall be quantified in z -, y - and $(z+y)$ - direction depending on the measuring system. For non-cartesian measuring systems, appropriate coordinates should be used.

For each case a sufficient number of calculations (≥ 20) should be done for different starting parameters for a random error and for different positions of a grid.

The whole method, englobing the measuring system and the smoothing and calculation procedure, can be accepted with reference to this leaflet, providing the measuring device can fulfil the accuracy determined by this validation procedure.

In Appendix G - page 87, some examples for acceptable and unacceptable results are given.

5 - Guidelines for application

This leaflet only gives a definition of equivalent conicity and the theoretical base for the calculation. No information is given about application.

This must be done in the appropriate leaflet (e.g. *UIC Leaflet 518*).

Nevertheless, some general recommendations should be made for applications:

1. Bending of the axle under the load cannot be ignored. Therefore the calculation must be made for normal service conditions.
2. For freight vehicles, contact geometry may be more significantly influenced by the load. If the equivalent conicity is to be used to investigate running behaviour, the contact geometry should be calculated in the empty and laden condition.
3. In normal service a vehicle will run on a very wide range of different rail profiles. It is impossible to take into consideration all rail profile types.
Generally, contact geometry and equivalent conicity calculations should be done using real wheel profiles in combination with design rail profiles. The rail inclination and the gauge must be taken into account.
This procedure is sufficient to determine the variation of the wheel profiles during a maintenance period.
4. In special cases (for example in cases of high instability, which cannot be explained by the contact geometry of the design profiles), the real rail profiles must be used for the calculation. If possible these measurements should be done with normal load on the track (e.g. by an inspection car), otherwise (especially on tracks with soft fastenings) the effects of the load may be taken into account in the interpretation of the results.
5. In the case of assessment of the equivalent conicity on the track, the real rail profile must be used for the calculation. In this case, real rail profile combinations will be made with the theoretical wheel profiles used on the line and/or typical real wheel profiles of vehicles running on the line.
6. The equivalent conicity is important for running at high speed on straight lines or in curves with very large radii.
On sharp curves the equivalent conicity is insignificant but the Δr function may influence the behaviour of the wheelset or the bogie. Because of the wear in such cases, the real rail profiles should be used.

Appendix A - Example of presentation of the Δr function and conicity

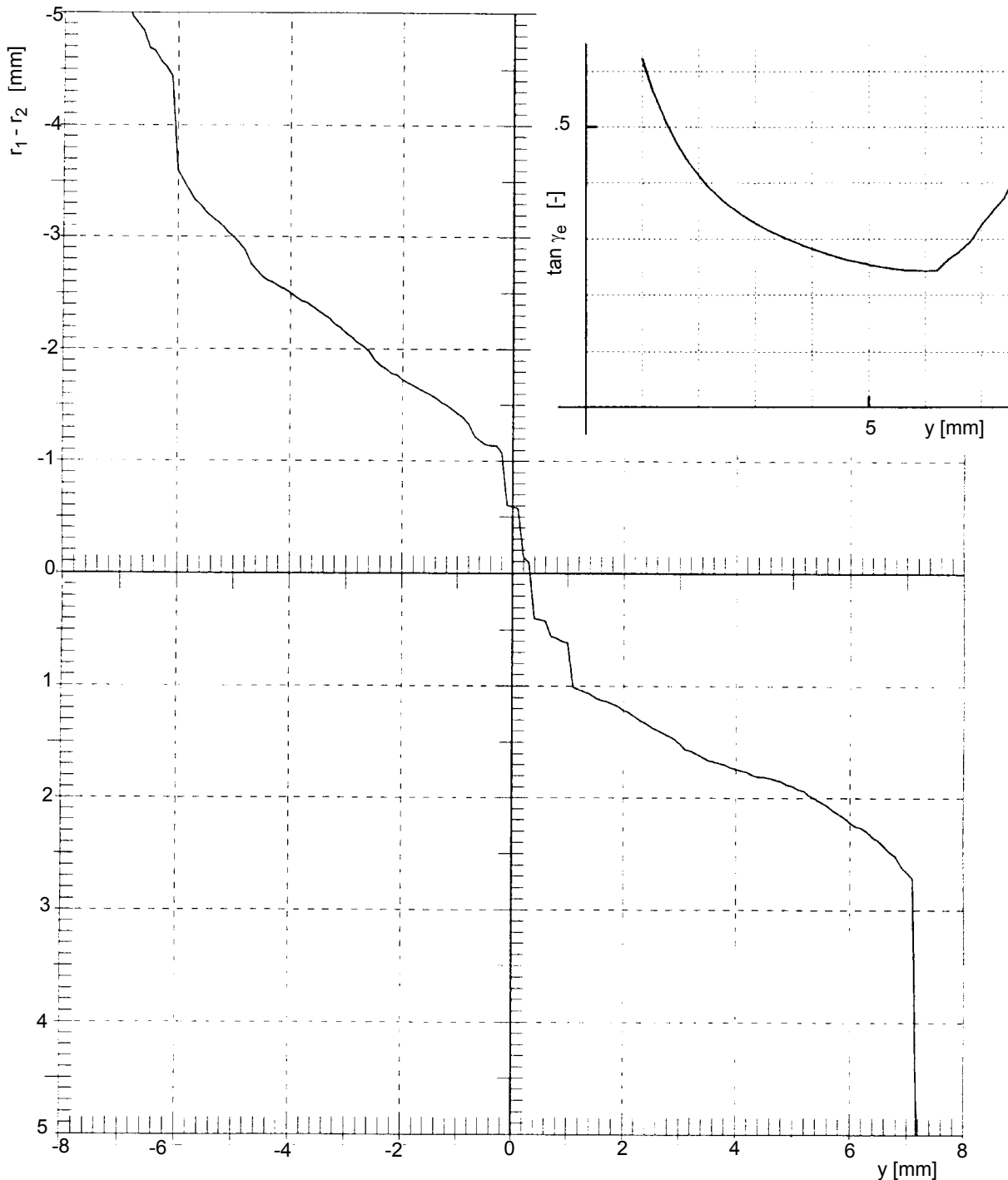


Fig. 1 -

$\Delta r = f(y)$ function and $\tan \gamma_e = f(y)$

Appendix B - Example of method for determining the equivalent conicity - Application of the KLINGEL formula

B.1 - Principle

The wheelset's movement on track can be formalised on the basis of angle:

$$\Psi = \frac{\dot{y}}{\dot{x}} = \frac{dy}{dx} \quad (1)$$

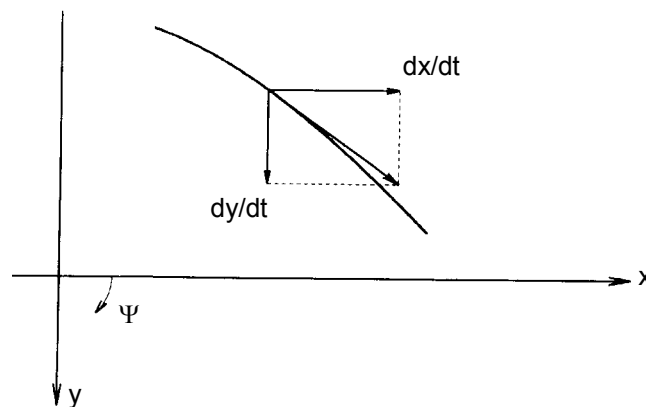


Fig. 2 -

Representation of dx , dy

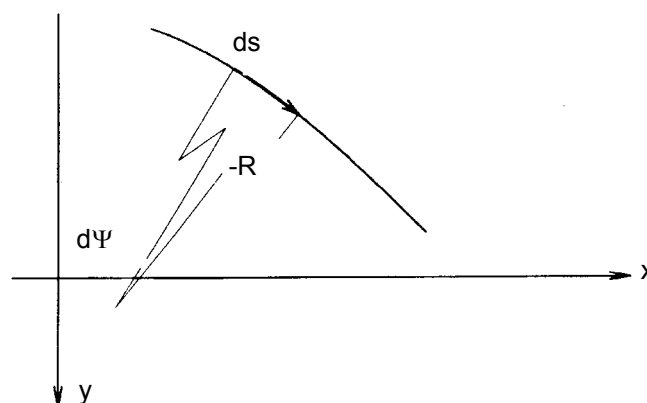


Fig. 3 -

Representation of ds , $d\Psi$

$$ds = -R d\Psi \quad (1)$$

for a small angle Ψ , $ds \cong dx$

$$dx = -R d\Psi \quad (2)$$

and, taking into account equation (1), this yields:

$$\Psi = \frac{dy}{dx} = -\frac{dy}{R d\Psi}$$

hence:

$$\Psi d\Psi = -\frac{dy}{R} \quad (3)$$

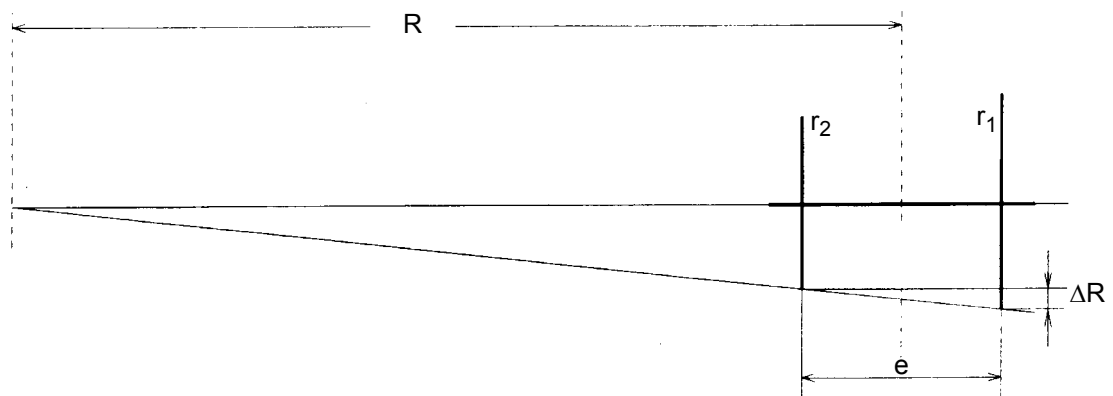


Fig. 4 -

Representation of r_0, r_1, r_2, e

$$\frac{R}{e} = \frac{\frac{r_1 + r_2}{2}}{r_1 - r_2}$$

where $\frac{r_1 + r_2}{2} = r_0$ is the nominal radius of each wheel

$$r_1 - r_2 = \Delta r$$

and e is the distance between the left and right contact points (approximately 1 500 mm).

Thus:

$$R = \frac{er_0}{\Delta r} \quad (4)$$

and, by replacing R in equation (3), we obtain:

$$\Psi \, d\Psi = \frac{-\Delta r dy}{er_0} \quad (5)$$

giving, by integration:

$$\frac{\Psi^2}{2} = -\frac{1}{er_0} [\int \Delta r dy + C] \quad (6)$$

This integration is used to obtain the movement of the wheelset on track and in particular the path corresponding to half of the wavelength starting from y_{emin} (for which $\Psi_{emin} = 0$) and rising to y_{emax} (for which $\Psi_{emax} = 0$).

The integral $\int \Delta r dy$ needs to be calculated only once for sufficiently large amplitudes to be able to cover the domain that will be needed for the remainder of the determination (Figure 7 - page 19).

The wheelset's movement on track is then obtained with the help of the following integration:

$$dx = \frac{1}{\Psi} dy \quad \text{or} \quad x = \int \frac{dy}{\Psi}$$

NB : equation (5) is equivalent to the differential equation $\frac{d^2 y}{dx^2} + \frac{\Delta r}{er_0} = 0$.

Indeed:

$$\Psi = \frac{dy}{dx}$$

$$\frac{d\Psi}{dx} = \frac{d^2 y}{dx^2}$$

The differential equation becomes:

$$\frac{d\Psi}{dx} + \frac{\Delta r}{er} = 0$$

with:

$$dx = \frac{dy}{\Psi}$$

whence:

$$\frac{\Psi}{dy} d\Psi + \frac{\Delta r}{er_0} = 0$$

such that:

$$\Psi d\Psi = -\frac{\Delta r dy}{er_0}$$

B.2 - Steps of the procedure

In practice this method is applied in the following way:

1. Based on the Δr function $= r_1 - r_2 = f(y)$, find the value y_{em} that corresponds to $\Delta r = 0$ (Figure 6 - page 18).
2. Calculate the $S(y) = -\int \Delta r dy$ function, starting from y_{em} in steps of $dy = + 0,1$ mm to $+ y$ and from y_{em} in steps of $dy = - 0,1$ mm to $-y$ (lower diagram in Figure 7 - page 19).
3. Determine corresponding amplitudes y_{emin} and y_{emax} and calculate the mean lateral movement $\hat{y}$ (Figure 7).
4. Find the functions $y_{emin} = f(\hat{y})$ and $y_{emax} = f(\hat{y})$ which allow determination, for a given lateral movement of the wheelset $2\hat{y}$, of the corresponding minimum and maximum amplitudes y_{emin} and y_{emax} (Figure 8 - page 19).
5. Compute the equivalent conicity $\tan \gamma_e$, for a given movement $\hat{y}$:
 - find the constant C of equation (6) (Figure 9 - page 20), such that $\Psi_{emin} = 0$ for the corresponding y_{emin} ;
 - calculate the angle Ψ by integrating equation (5) to give:

$$\Psi = \sqrt{\frac{-2}{er_0} [\int \Delta r dy + C]}$$

in steps of $dy = 0,1$ mm;

- calculate the abscissa of the wheelset movement:

$$x = f(y) = \int \frac{dy}{\Psi} \quad \text{between } y_{emin} \text{ and } y_{emax}$$

which allows determination of wavelength λ of the wheelset's kinematic motion.

In most cases, the integration cannot be done in only one step in the range from y_{emin} to y_{emax} . Therefore, wavelength x must be calculated by adding up $dx = \frac{dy}{\Psi}$, whereby the step of dy should be $\leq 0,1$ mm;

- calculate the equivalent conicity, applying the Klingel formula:

$$\tan \gamma_e = \left(\frac{\pi}{\lambda} \right)^2 2er_0$$

- Determine function $\tan \gamma_e = f(\hat{y})$

by applying paragraph 5 - page 16, for $\hat{y}$ amplitudes starting at 1 mm up to the maximum permitted by the $\Delta r = f(y)$ characteristic, with a maximum step $\Delta \hat{y} = 0,5$ mm.

B.3 - Particularities

$\Delta r = f(y)$ **characteristic with a negative slope**

- When the $\Delta r = f(y)$ characteristic has a negative slope, it is necessary to select the most suitable point for y_{em} .

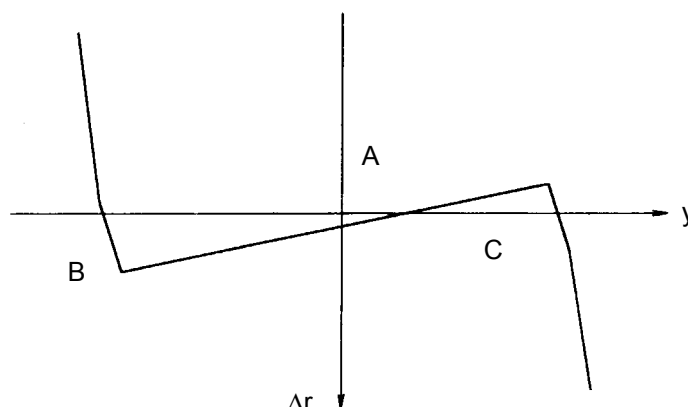


Fig. 5 -

$\Delta r = f(y)$ *characteristic with negative slope*

In the example illustrated in the figure, instead of choosing Point A for y_{em} , choose Point B or Point C according to the initial position of the wheelset.

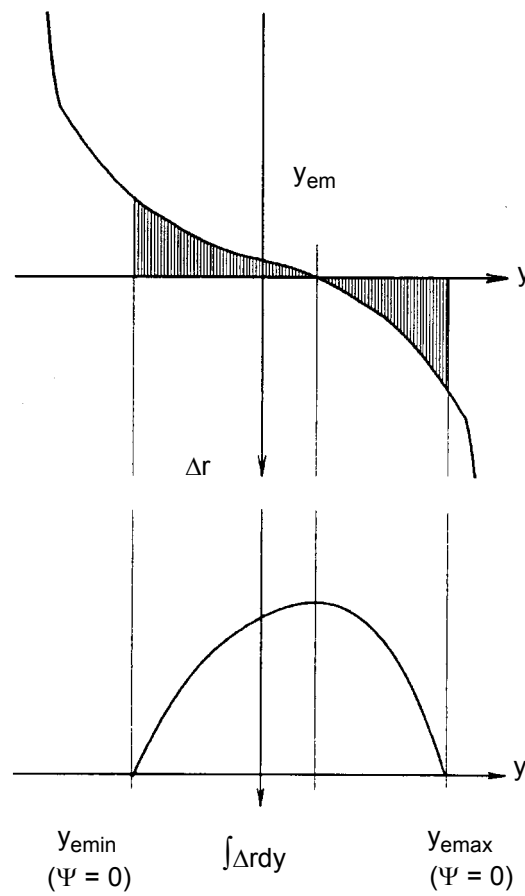
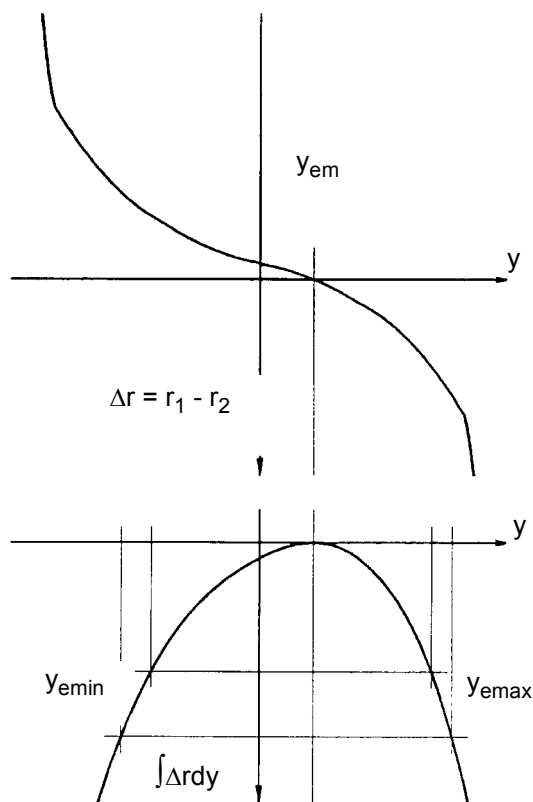


Fig. 6 -

Calculation of $\int \Delta r dy$ integral



Determination of $y_{em} : \Delta r = 0$

Calculation:

$$\int_{y_{em}}^{+y} \Delta r dy = f(y)$$

$$\int_{y_{em}}^{-y} \Delta r (-dy) = f(y)$$

Determination:

corresponding y_{emin}, y_{emax}

$$\text{and } \hat{y} = (y_{emax} - y_{emin})/2$$

Fig. 7 -

Determination of y_{em} , calculation of $\int \Delta r dy$ and determination of $\hat{y}$

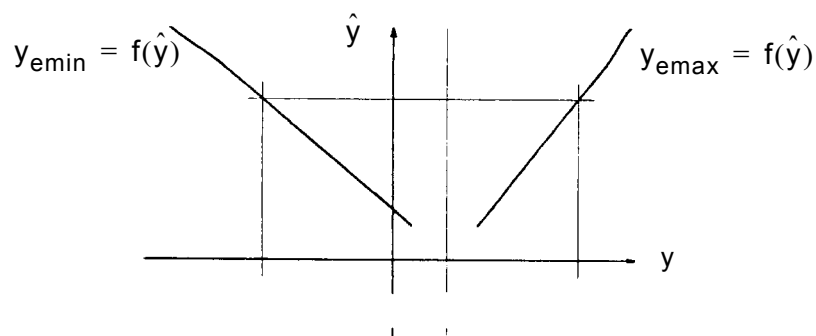


Fig. 8 -

Determination of $y_{emin} = f(\hat{y})$ and $y_{emax} = f(\hat{y})$ functions

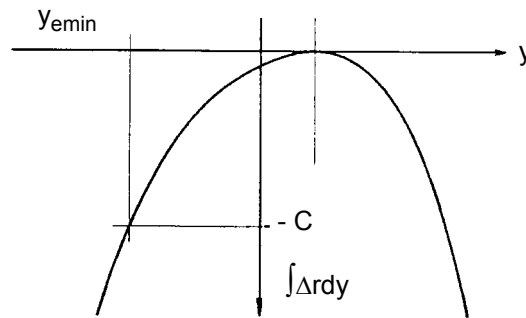


Fig. 9 -

Determination of C constant

Appendix C - Example of method for determining the equivalent conicity - Linear regression of the Δr function

C.1 - Principles

For a linear Δr function, the slope of the Δr function is equal to $2 \tan \gamma_e$.

For a non-linear Δr function, the slope of a linear regression gives an approximation of $2 \tan \gamma_e$.

For an interval of $2\hat{y}$, the regression of the Δr function must be done between y_{emin} and y_{emax} , where y_{emin} and y_{emax} are the minimum and maximum amplitude of the real shape of the wave.

For symmetrical Δr functions, y_{emin} and y_{emax} are equal to mean amplitude $\hat{y}$, but for non-symmetrical Δr functions, y_{emin} and y_{emax} normally are unequal to $\hat{y}$.

For a mean amplitude $\hat{y}$, the minimum and maximum amplitudes y_{emin} and y_{emax} can be calculated by the Δr function as described in Appendix B - page 13.

C.2 - Steps of the procedure

In practice, this method is applied in the following way:

1. Based on the function $\Delta r = r_1 - r_2 = f(y)$, find the value y_{em} that corresponds to $\Delta r = 0$ (Figure 6 - page 18).
2. Calculate the $S(y) = -\int \Delta r dy$ function, starting from y_{em} in steps of $dy = +0,1$ mm to $+y$ and from y_{em} in steps of $dy = -0,1$ mm to $-y$ (lower diagram in Figure 7 - page 19).
3. Determine the corresponding amplitudes y_{emin} and y_{emax} and calculate the mean lateral movement $\hat{y}$ (Figure 7).
4. Find the functions $y_{emin} = f(\hat{y})$ and $y_{emax} = f(\hat{y})$ which allow determination, for a given lateral movement of wheelset $2\hat{y}$, of the corresponding minimum and maximum amplitudes y_{emin} and y_{emax} (Figure 8 - page 19).
5. Calculate the linear regression of the Δr function in the range between y_{emin} and y_{emax} and calculate the equivalent conicity by $\tan \gamma_e = \frac{B}{2}$ where B = slope of the regression.
6. Determine the function $\tan \gamma_e = f(\hat{y})$

by applying paragraph 5, for $\hat{y}$ amplitudes starting at 1 mm up to the maximum permitted by the $\Delta r = f(y)$ characteristic, with a maximum step $\Delta \hat{y} = 0,5$ mm.

C.3 - Particularities

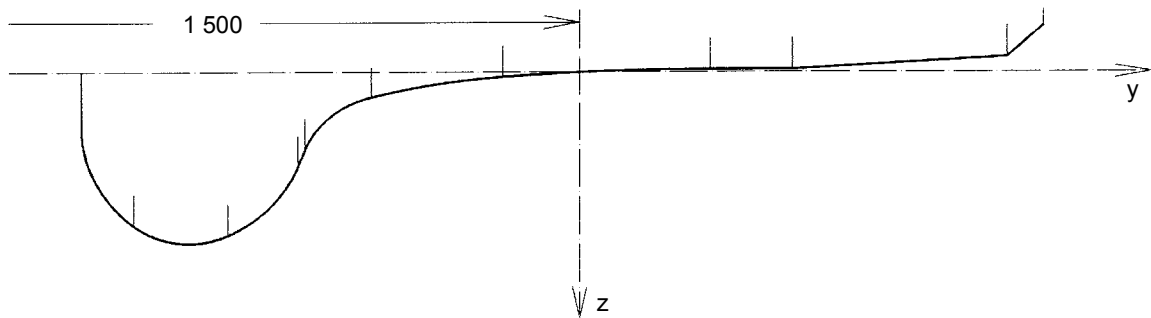
If the $\Delta r = f(y)$ characteristic presents a substantial asymmetry around the origin over the $2\hat{y}$ interval, which can be likened to two straight line segments, it is recommended applying a regression to each of the parts (positive and negative) to obtain two conicities $\tan\gamma_{e,p}$ and $\tan\gamma_{e,n}$. The equivalent conicity can then be obtained by:

$$\tan\gamma_e = \left[\frac{2}{\frac{1}{\sqrt{\tan\gamma_{e,p}}} + \frac{1}{\sqrt{\tan\gamma_{e,n}}}} \right]^2$$

Appendix D - Reference profiles

D.1 - Wheel A

D.1.1 - Drawing



D.1.2 - Analytical definition

z coordinates of the profile are defined in the following ranges:

From y = -70	to y = -62.7647	:	$z = +9.5193 + (20.5^2 - (y + 49.5)^2)^{0.5}$
From y = -62.7647	to y = -49.6625	:	$z = +16 + (12^2 - (y + 55)^2)^{0.5}$
From y = -49.6625	to y = -39.7645	:	$z = +8.8349 + (20^2 - (y + 58.5583)^2)^{0.5}$
From y = -39.7645	to y = -38.7372	:	$z = -93.5767 - 2.7475 \cdot y$
From y = -38.7372	to y = -29.4591	:	$z = +17.6421 - (14^2 - (y + 25.5816)^2)^{0.5}$
From y = -29.4591	to y = -10.8749	:	$z = +97.16492 - (96.7605^2 - (y + 2.6594)^2)^{0.5}$
From y = -10.8749	to y = +18.4173	:	$z = +344.5081 - (345^2 - (y - 18.4173)^2)^{0.5}$
From y = +18.4173	to y = +30	:	$z = -0.4919 + 0 \cdot y$
From y = +30	to y = +60	:	$z = +1.5091 - 0.0667 \cdot y$
From y = +60	to y = +65	:	$z = +57.5071 - 1 \cdot y$

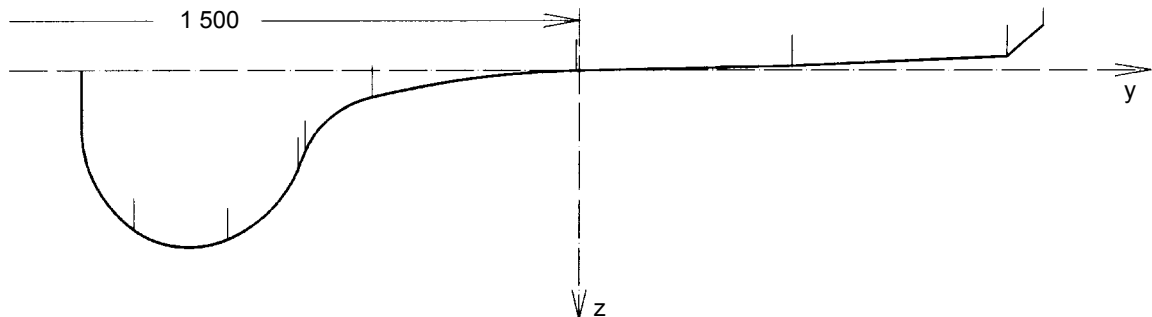
D.1.3 - Cartesian coordinates

Wheel profile: R-UIC519-A
Right wheel

y [mm]	z [mm]	y [mm]	z [mm]	y [mm]	z [mm]
-70.000	9.519	-20.000	1.971	30.000	-.492
-69.000	15.844	-19.000	1.794	31.000	-.559
-68.000	18.351	-18.000	1.628	32.000	-.625
-67.000	20.196	-17.000	1.473	33.000	-.692
-66.000	21.685	-16.000	1.328	34.000	-.759
-65.000	22.936	-15.000	1.195	35.000	-.825
-64.000	24.011	-14.000	1.071	36.000	-.892
-63.000	24.947	-13.000	.959	37.000	-.959
-62.000	25.747	-12.000	.856	38.000	-1.026
-61.000	26.392	-11.000	.765	39.000	-1.092
-60.000	26.909	-10.000	.680	40.000	-1.159
-59.000	27.314	-9.000	.599	41.000	-1.226
-58.000	27.619	-8.000	.521	42.000	-1.292
-57.000	27.832	-7.000	.446	43.000	-1.359
-56.000	27.958	-6.000	.373	44.000	-1.426
-55.000	28.000	-5.000	.304	45.000	-1.492
-54.000	27.958	-4.000	.237	46.000	-1.559
-53.000	27.832	-3.000	.174	47.000	-1.626
-52.000	27.619	-2.000	.113	48.000	-1.693
-51.000	27.314	-1.000	.055	49.000	-1.759
-50.000	26.909	0.000	0.000	50.000	-1.826
-49.000	26.403	1.000	-.052	51.000	-1.893
-48.000	25.821	2.000	-.101	52.000	-1.959
-47.000	25.157	3.000	-.147	53.000	-2.026
-46.000	24.401	4.000	-.191	54.000	-2.093
-45.000	23.538	5.000	-.231	55.000	-2.159
-44.000	22.548	6.000	-.268	56.000	-2.226
-43.000	21.402	7.000	-.303	57.000	-2.293
-42.000	20.052	8.000	-.335	58.000	-2.360
-41.000	18.411	9.000	-.363	59.000	-2.426
-40.000	16.291	10.000	-.389	60.000	-2.493
-39.000	13.576	11.000	-.412	61.000	-3.493
-38.000	11.178	12.000	-.432	62.000	-4.493
-37.000	9.541	13.000	-.449	63.000	-5.493
-36.000	8.290	14.000	-.464	64.000	-6.493
-35.000	7.284	15.000	-.475	65.000	-7.493
-34.000	6.456	16.000	-.483		
-33.000	5.769	17.000	-.489		
-32.000	5.200	18.000	-.492		
-31.000	4.733	19.000	-.492		
-30.000	4.358	20.000	-.492		
-29.000	4.059	21.000	-.492		
-28.000	3.782	22.000	-.492		
-27.000	3.516	23.000	-.492		
-26.000	3.262	24.000	-.492		
-25.000	3.019	25.000	-.492		
-24.000	2.787	26.000	-.492		
-23.000	2.567	27.000	-.492		
-22.000	2.357	28.000	-.492		
-21.000	2.159	29.000	-.492		

D.2 - Wheel B

D.2.1 - Drawing



D.2.2 - Analytical definition

z coordinates of the profile are defined in the following ranges:

From y = -70	to y = -62.7647	:	$z = +9.5193 + (20.5^2 - (y + 49.5)^2)^{0.5}$
From y = -62.7647	to y = -49.6625	:	$z = +16 + (12^2 - (y + 55)^2)^{0.5}$
From y = -49.6625	to y = -39.7645	:	$z = +8.8349 + (20^2 - (y + 58.5583)^2)^{0.5}$
From y = -39.7645	to y = -38.7879	:	$z = -93.5767 - 2.7475 \cdot y$
From y = -38.7879	to y = -29.3505	:	$z = +17.7814 - (14^2 - (y + 25.6322)^2)^{0.5}$
From y = -29.3505	to y = -0.4787	:	$z = +119.9745 - (120^2 - (y - 2.5204)^2)^{0.5}$
From y = -0.4787	to y = +30	:	$z = +0 - 0.025 \cdot y$
From y = +30	to y = +60	:	$z = +0.75 - 0.05 \cdot y$
From y = +60	to y = +65	:	$z = +57.75 - 1 \cdot y$

D.2.3 - Cartesian coordinates

Wheel profile: R-UIC519-B
Right wheel

y [mm]	z [mm]	y [mm]	z [mm]	y [mm]	z [mm]
-70.000	9.519	-20.000	2.107	30.000	-.750
-69.000	15.844	-19.000	1.920	31.000	-.800
-68.000	18.351	-18.000	1.742	32.000	-.850
-67.000	20.196	-17.000	1.573	33.000	-.900
-66.000	21.685	-16.000	1.412	34.000	-.950
-65.000	22.936	-15.000	1.260	35.000	-1.000
-64.000	24.011	-14.000	1.117	36.000	-1.050
-63.000	24.947	-13.000	.982	37.000	-1.100
-62.000	25.747	-12.000	.856	38.000	-1.150
-61.000	26.392	-11.000	.739	39.000	-1.200
-60.000	26.909	-10.000	.629	40.000	-1.250
-59.000	27.314	-9.000	.529	41.000	-1.300
-58.000	27.619	-8.000	.437	42.000	-1.350
-57.000	27.832	-7.000	.353	43.000	-1.400
-56.000	27.958	-6.000	.277	44.000	-1.450
-55.000	28.000	-5.000	.210	45.000	-1.500
-54.000	27.958	-4.000	.152	46.000	-1.550
-53.000	27.832	-3.000	.102	47.000	-1.600
-52.000	27.619	-2.000	.060	48.000	-1.650
-51.000	27.314	-1.000	.026	49.000	-1.700
-50.000	26.909	0.000	0.000	50.000	-1.750
-49.000	26.403	1.000	-.025	51.000	-1.800
-48.000	25.821	2.000	-.050	52.000	-1.850
-47.000	25.157	3.000	-.075	53.000	-1.900
-46.000	24.401	4.000	-.100	54.000	-1.950
-45.000	23.538	5.000	-.125	55.000	-2.000
-44.000	22.548	6.000	-.150	56.000	-2.050
-43.000	21.402	7.000	-.175	57.000	-2.100
-42.000	20.052	8.000	-.200	58.000	-2.150
-41.000	18.411	9.000	-.225	59.000	-2.200
-40.000	16.291	10.000	-.250	60.000	-2.250
-39.000	13.576	11.000	-.275	61.000	-3.250
-38.000	11.221	12.000	-.300	62.000	-4.250
-37.000	9.610	13.000	-.325	63.000	-5.250
-36.000	8.373	14.000	-.350	64.000	-6.250
-35.000	7.377	15.000	-.375	65.000	-7.250
-34.000	6.557	16.000	-.400		
-33.000	5.877	17.000	-.425		
-32.000	5.313	18.000	-.450		
-31.000	4.851	19.000	-.475		
-30.000	4.480	20.000	-.500		
-29.000	4.188	21.000	-.525		
-28.000	3.921	22.000	-.550		
-27.000	3.662	23.000	-.575		
-26.000	3.413	24.000	-.600		
-25.000	3.173	25.000	-.625		
-24.000	2.942	26.000	-.650		
-23.000	2.720	27.000	-.675		
-22.000	2.506	28.000	-.700		
-21.000	2.302	29.000	-.725		