



ABC OF PHYSICS

A Very Brief Guide

Lev Okun

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Lev Okun

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Preface

0.1 For whom this book is written*

Physics is the science describing nature (*physis* in Greek ($\varphi\upsilon\sigma\iota\varsigma$) means nature), and this book concentrates on its foundations (its “ABC”). The foundation of the physics of the 21st century is the so-called Standard Model of elementary particles of which everything that exists is built in accordance with the laws of the theory of relativity and quantum theory.

It is regrettable that the existing introductions to these sciences are beyond the grasp not only of students but also of the majority of physics teachers, all because of the mathematical complexity of the available textbooks. This little book is a result of my attempt to present briefly and with maximum simplicity the fundamentals of the relativity theory (with its main constant c) and quantum mechanics (with its main constant $\hbar$). I do understand, of course, that I cannot write a popular book which would be comprehensible to readers knowing no physics. Consequently, this book is intended first and foremost for pro-

fessional physicists (especially for young professors) in the hope that some of them would use the ideas presented in the book for writing simple courses of modern physics and science-popularising books and articles.

In contrast to popular science books, this one makes extensive use of mathematical formulas. However, in contrast to the formulas in university-level textbooks, formulas selected for this book are so simple that the knowledge of elementary mathematics taught at high schools is sufficient for understanding them. I am convinced therefore that many of the pages¹ of the book will be understood even by very young people and should help those who are attracted to physics to gradually expand their scientific horizons. To achieve this, the book ends with a detailed index. There is no doubt that the small size of this brief guide excludes using it as a replacement for a proper textbook.

Elementary particles of matter moving at speeds close to speed of light c and at very small values of angular momentum approaching Planck's quantum $\hbar$, have very unusual properties. However, it is precisely their unusual, counterintuitive nature that makes the world around us what it is.

One of the best scientific descriptions of the world, based on the theory of relativity and quantum mechanics is found in the fundamental monographs written by Stephen Weinberg [1],[2],[3]. However, to present the most essential

¹The corresponding sections and chapters are marked by an asterisk.

elements of these sciences in this small and simplified book, I not only had to select the simplest of formulas but also to refuse mentioning many complex concepts that make the subjects of these and many other treatises of modern physics.

0.2 On the contents of some of the chapters

The fundamental concept of relativity theory is the relation between energy E , momentum $\mathbf{p}$ and mass m of a free particle. this relation, explained in detail in chapter 4, makes it possible to understand within the same framework the properties both of massless particles, such as particle of light — photon, and of particles with nonzero mass, such as electron. According to quantum mechanics, the masses of all particles of one type, e.g. the masses of all electrons in the universe, are strictly identical.

Alas, the equality $E = mc^2$, ubiquitous in popular science literature and interpreted by some as the dependence of the particle mass m on particle velocity v , promotes a false idea of what mass actually is. This false notion gets entrenched even further when people refer to the genuine mass m as “rest mass” and denote it by m_0 , thus breaking the link between the theory of relativity and Newtonian mechanics, which is the limiting case of the theory of relativity for low velocities.

The spin (i.e. the intrinsic (proper) angular momentum of a particle) plays the fundamental role in the workings of Nature. (I will explain in chapter 5 that the magnitude

of spin is usually expressed in units of $\hbar$.) Particles whose spin is an integer multiple of $\hbar$ are known as bosons. Particles whose spin is a half-integer multiple of $\hbar$ are known as fermions. The behavior of particles in a system consisting of two or greater number of fermions is radically different from the behavior of particles in a system consisting of two or greater number of bosons. For instance, the properties of atoms in the periodic table of elements are a consequence of the electron spin being equal to $\frac{1}{2}\hbar$.

The photon's spin equals $1\hbar$. Electrons interact with each other and with other electrically charged particles by exchanging photons. The theory of interaction of photons with electrons is called Quantum Electrodynamics (QED). QED is the most advanced physics theory, and has been confirmed experimentally to within ten significant digits. Likewise, the exchange of gravitons — massless particles with spin $2\hbar$ — explains gravitation.

In contrast to the approach favored by most textbooks on quantum mechanics, the basis of the presentation in this book is not the relatively complex concept of the wave function of a physical system (such as the atom) but a much simpler concept of an elementary quantum state of the system (atom's energy level) which is characterized by the values of several quantum numbers. See chapter 5.

You will see in chapter 6 that this makes it possible to introduce the amplitude of transition between two quantum states. The amplitude modulus squared gives the probability of transition (decay of an excited state of the

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atom resulting in the emission of a photon) as a function of time for an ensemble of identical states. Note that the decay of a particular atom is a completely random process, and the moment of time when this decay will occur cannot be predicted in principle. This approach eliminates many of the difficulties of interpretation of quantum mechanics involved in the problem of measurement (reduction of the wave function). On the other hand, this approach makes it possible to highlight those quantum aspects which are not related to the procedure of measurement, and thereby go beyond the confines of defining quantum mechanics as a science which only establishes relations between the results of measurements.

Another specific feature of this book is that the conventional gravitational interaction is treated as a quantum process of exchanging a massless particle of spin 2 known as graviton. Newton's constant G_N together with the constants $\hbar$ and c define the so-called Planck mass $m_P = (\hbar c/G_N)^{\frac{1}{2}} \approx 2.2 \cdot 10^{-5}$ gram. The theory that would describe the gravitational interactions at energies on the order of $m_P c^2$ has not been worked out so far although the correctness of descriptions of gravitation as exchange of gravitons has been confirmed by experiments at all attainable energies. See chapter 16.

0.3 Two beacons*

I wrote this book in the hope that it will prove useful to young people and will help them find their place in

this rapidly changing and in many ways incomprehensible world. The main idea I wish to communicate is that the truth does exist and that one can get much closer to it than many people think. The physics holds several simple and inviolable truths that any thinking person can understand. Mankind was able to uncover these truths when studying nature, mostly during the last hundred years.

At first glance, the science has grown, ramified and specialized so much that a non-expert simply cannot hope to understand what its foundations are. The rapid growth of the Internet exposes us to an unprecedented flow of information and misinformation. It is impossible to figure out what is what unless one understands the physical meaning of the constants c and $\hbar$. These two universal constants will help you, like two guiding stars, to find your way to understanding nature. Choosing these most fundamental constants of nature to serve as your guiding stars in science will allow you to gradually find like-minded individuals and to formulate criteria of truth in specific situations.

Newton compared himself to a boy playing with pebbles on the shore of the great ocean of truths waiting to be discovered. Physics has expanded enormously since then and the area of the known has become vast. Some contemporary scientists are convinced that the realm of the unknown shrank to become smaller than that of the known. I do not know if this is true but I consider it extremely important to separate what has been established with certainty from what only looks plausible.

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It seems to me very important to separate the well-established and understood phenomena and properties (such as for example the properties of atoms and their nuclei or the main properties of the solar system) from much more complex issues (such as the birth of the Universe or the behavior of the gravitational interaction at the Planck scale where it is stronger than all other interactions). Such theories as classical mechanics and electrodynamics, or quantum mechanics and field theory will be valid forever. Only certain improvements are possible that would not change the established mathematical relations between physical quantities but would clarify the range of applicability of these relations and justify the choice of the variables they have introduced. In contrast to this, far-fetched theoretical extrapolations may not be confirmed by subsequent measurements.

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Lev Okun

Moscow

February 2012

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Chapter 1

The Fundamentals

1.1 On intuition

The purpose of this book is to explain within a hundred-odd pages how modern elementary particle physics allows us to understand the workings of the surrounding world. Two universal constants lie at the basis of the cosmos: the maximum speed of uniform rectilinear motion c and minimum quantum of rotational and oscillatory motion $\hbar$. The words “rectilinear” and “uniform” seem to point to unbounded, infinite motion through space and to linearity of the time (the philosophy of the Occident). Rotation points to cyclical nature of time and to bounded, finite nature of motion (the philosophy of the Orient).

The matter I wish to present here is not philosophy but the way in which physicists quantitatively describe their observations and experiments using intuition as their tool. The main problem of modern physics is that the concepts of maximum speed and minimum quantum appear extremely anti-intuitive to those who do not work in fundamental physics.

1.2 Space and time*

Our space is three-dimensional: any observer can imagine a system of coordinates — three mutually orthogonal axes: back-to-forward (x), left-to-right (y), bottom-to-top (z). The observer typically places himself at the origin of coordinates.

Imagine an observer following the motion of a particle. He characterizes the position of the particle at the moment of time t by the radius vector $\mathbf{r}$ with its three coordinates: x, y, z . The coordinate system plus time form what we call the frame of reference. All points in space and time are equivalent: both space and time are uniform. In addition, space is isotropic: the three axes can be oriented in an arbitrary manner. Note that under such re-orientations the magnitude of the radius vector remains unchanged: $r \equiv |\mathbf{r}| = \text{const.}$

1.3 Matter and substance*

Any change in the motion or behavior of a particle is a manifestation of elementary (i.e. the smallest and indivisible) particles of matter. We begin with three elementary particles of matter: the photon γ , the electron e and the proton p . We will first see how the simplest hydrogen atom emerges as a result of their interaction. We will then look at other atoms so as to understand how the matter around us emerges as a result of their interaction and formation of increasingly complex systems of interlinked particles.

Then we will go deeper step by step and learn about all other elementary particles.

There is still no universally accepted definition of what should be called ‘substance’ and what should be called ‘matter’. Many authors use the term “material particle” only if the particle has nonzero mass. Suffice it to recall the term “material point” which is used in the literature by many authors (see e.g. Weinberg’s book [1]) where they discussed “massive points”, that is bodies whose size can be considered negligible in a specific problem. For these authors photons are particles of radiation, not of matter. The debate on what should be called which name will be solved some day. At the moment I will refer to all particles including photons as particles of matter. And I will call atoms and everything built of atoms as substance.

1.4 Motion*

Motion is the displacement of a particle in space. If the motion is translational (i.e. uniform and rectilinear) then the velocity $\mathbf{v}$ of displacement is given by the formula $\mathbf{v} = \mathbf{r}/t$ where $\mathbf{r}$ is the path covered and t is the time the displacement took. The definition of velocity in the case of arbitrary motion will be given a little later.

Chapter 2

Units*

2.1 Standards

Every measurement is a comparison of what is being measured with a known standard. The generally accepted standards with which results of measurements are compared are called units.

2.2 Circle and angles

Angles are measured in degrees (minutes and seconds) or in radians. One degree of arc is the angle subtended at the center of a circle by the arc obtained by dividing the circle into 360 equal parts. The minute of arc: $1' = 1^\circ/60$. The second of arc: $1'' = 1'/60$. We know that the ratio of the circumference of the circle to its radius equals 2π , where $\pi \approx 3.14$. One radian $1 \text{ rad} = 360^\circ/2\pi \approx 57^\circ$ is the angle subtended by the arc whose length equals that of the radius.

2.3 Units of time and length

Units of time are determined by the rotation of the Earth. A day and a night are 24 hours long. One hour = 60 min-

utes. One minute = 60 seconds. Although one hour and one degree of arc consist of 60 minutes and 3600 seconds, this is the only common feature between the units of angle and time.

Lengths are measured by comparing them against the established length standard. The meter is the typical unit for measuring lengths.

Chapter 3

A Minimum of Mathematics

Mathematics is often said to be the queen and a servant of all the natural sciences. But mathematics is also a powerful obstacle when people are introduced to these sciences. This is why authors of popular science books and articles avoid using mathematical notation. In my opinion, modern physics cannot be explained without mathematics. I will try to limit myself to a minimum of mathematical knowledge. But this minimum is necessary and unavoidable.

3.1 Four operations of school mathematics and the imaginary unit*

Two types of arithmetic operations will be needed here to reach understanding of the laws of nature: 1) addition/subtraction, 2) multiplication/division, and two types of algebraic operations: 3) raising to power and 4) finding the logarithm. It won't be soon that we will need this last operation. We will also need the concept of the imaginary unit $i \doteq \sqrt{-1}$ and of complex numbers.

3.2 Powers of ten*

When studying nature, we constantly come across very large or very small numbers which are conveniently presented as powers of ten. Thus the speed of light is $3 \cdot 10^5$ km/s = $3 \cdot 10^8$ m/s, and the size of an atom is on the order of 10^{-10} m. Unlike authors of popular science books, I will not use expressions like “one thousandth of one millionth of”; I will write 10^{-9} .

3.3 Prefixes of the powers of ten

Many physical terms include the following prefixes:

Deca	10^1	da	Deci	10^{-1}	d
Hecto	10^2	h	Centi	10^{-2}	c
Kilo	10^3	k	Milli	10^{-3}	m
Mega	10^6	M	Micro	10^{-6}	μ
Giga	10^9	G	Nano	10^{-9}	n
Tera	10^{12}	T	Pico	10^{-12}	p
Peta	10^{15}	P	Femto	10^{-15}	f
Exa	10^{18}	E	Atto	10^{-18}	a
Zetta	10^{21}	Z	Zepto	10^{-21}	z
Yotta	10^{24}	Y	Yocto	10^{-24}	y

3.4 Differentiation and integration

The velocity of arbitrary motion is determined as the derivative of distance with respect to time: $\mathbf{v} = d\mathbf{r}/dt$. The quantity dt stands for a sufficiently small (“infinitely

small”) difference between two very close points in time. The quantity $d\mathbf{r}$ has a similar meaning — the difference between two points in space very close to one another. Here the infinitely small distance between two points in space is divided by the infinitely short interval of time, which gives as a result the finite velocity of motion. This operation is called differentiation. Integration is an operation inverse to differentiation: $\mathbf{r} = \int \mathbf{v} dt$. We won’t need these operations just yet. It would be very useful to readers — both familiar with the concepts of differentiation and integration and to complete novices — to browse time and again the remarkable book written by Yakov Borisovich Zeldovich [4].

3.5 Matrices

To learn about quantum mechanics, it will be necessary for us to be introduced to matrices.

Chapter 4

Translational Motion*

4.1 Free particle

The notion of free body or free particle is undoubtedly an idealization — an abstraction, but an exceptionally fruitful abstraction. A particle is described as free if it is not subjected to external forces. If it was at rest then it will continue to be at rest; if it was moving at some velocity, it will keep moving at the same velocity in the same direction. Ordinary bodies around us do stop after a while but this happens because they are not free: other bodies act on them as brakes.

4.2 Maximum velocity c

A remarkable result achieved during the last two-three centuries was the discovery of the fact that nature imposes a maximum on the velocity of translational motion in vacuum, equal to the speed of light $c \approx 300,000$ km/s (to be precise, $c = 299\,792\,458$ m/s; in fact this equality creates a link between the definitions of meter and second).

4.3 Energy and momentum of a particle

The uniform linear motion — i.e. the motion of a free particle — is characterized by two quantities that are conserved in time: its energy E and its momentum $\mathbf{p}$. Energy is a scalar quantity and momentum is a vector quantity characterized by a direction in space. Energy conservation is a consequence of time uniformity while conservation of momentum is a consequence of space uniformity.

Energy is generally defined as the universal measure of all processes and the momentum as the universal measure of all motions. However, these definitions are not specific enough for one to work with them. The evolution of mathematics and physics during the last century was such that the definitions of basic concepts were becoming progressively more difficult while the operations over them and the proofs were getting progressively easier. Energy and momentum are excellent examples of this. It appears that the optimal way of working with them lies in writing the basic formulas for them and gradually finding out how they agree with the concepts familiar to the reader. We shall start with Newton's mechanics.

4.4 Kinetic and potential energy in Newtonian mechanics

The kinetic energy of a free body in Newtonian mechanics is defined as the quantity E_K :

$$E_K = m\mathbf{v}^2/2 = \mathbf{p}^2/2m. \quad (4.1)$$

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If the body is not free but exists in some sort of force field then it possesses a potential energy U in addition to the kinetic energy E_K . For example, a mug on the table has potential energy greater than its potential energy on the floor by a quantity $U = mgh$ where m is the mug's mass, $g \approx 10 \text{ m/s}^2$ is the gravity acceleration on the Earth, and h is the height of the table. When a body falls, its potential energy is converted into kinetic energy so that the sum of the two energies $E = U + E_K$ remains unchanged. Likewise the tiniest electrically charged particle — an electron — moving across a potential difference of 1 Volt changes its kinetic energy by one electron-volt: 1 eV.

4.5 Momentum in Newtonian mechanics

The term 'momentum' carries many different meanings, of which two are important for us here: 1) a short-duration push setting a body in motion, 2) a vector quantity $\mathbf{p}$ possessing three components p_x , p_y , p_z and characterizing the moving body. Galileo and Newton were first to understand that a free stand-alone body receiving momentum would in empty space move infinitely long along a straight line at constant velocity. (The fact that the momentum of bodies in our everyday life is not conserved and bodies do not move infinitely long but stop earlier or later is caused by the action of other bodies on them.)

In Newtonian mechanics the momentum of a body is related to its velocity by the formula $\mathbf{p} = m\mathbf{v}$.

The position of a body $\mathbf{r}'$ at time t is connected to its position $\mathbf{r}$ at time $t = 0$ via the Galilean transformation¹:

$$\mathbf{r}' = \mathbf{r} + \mathbf{v}t \quad (4.2)$$

It is obvious that this transformation stems from the tacit assumption that time t is independent of the reference frame. You will see below that it is very easy to demonstrate that this assumption contradicts the existence of the limiting velocity in nature.

Indeed, if the same initial momentum is imparted to a 10 times lighter body, its velocity would be 10 times higher and it would cover 10 times longer path over the same interval of time.

We shall now repeat the operation of reducing the mass of the particle by a factor of 10. Now its velocity in Newtonian mechanics would have grown 100 times. In Newtonian mechanics, this operation can be performed an unlimited number of times with the same tenfold increase in speed. We recall now that nature has this limiting velocity: $c = 3 \cdot 10^8$ m/s. Then the increase in velocity should begin to decline long before we approach this limit. The increment to kinetic energy must also change. We have to conclude that Newtonian mechanics should be replaced by

¹Here I mean an active transformation when the position of the object undergoes changes, not when the coordinate system does.

some other mechanics. It is called the relativistic mechanics or Einstein's relativity theory. Scientists who successively made key contributions to it were Lorentz, Poincaré, Einstein and Minkowski. Newtonian mechanics (nonrelativistic mechanics) is a special case of the theory of relativity for situations when bodies move at velocities small compared to the speed of light.

4.6 Space and time in relativistic mechanics

To recapitulate: when the velocity of an object approaches the maximum speed c , nonrelativistic Newtonian mechanics becomes completely inapplicable and must be replaced with relativistic mechanics.

Galilean coordinate transformation has been replaced by the Lorentz transformation:

$$\begin{aligned} \mathbf{r}' &= \gamma(\mathbf{r} + \mathbf{v}t) \\ t' &= \gamma(t + \mathbf{r}\mathbf{v}/c^2), \end{aligned} \quad (4.3)$$

where

$$\gamma = 1/\sqrt{1 - v^2/c^2}. \quad (4.4)$$

It is easy to see that as $v/c \rightarrow 0$, the Lorentz transformation changes into the Galilean transformation for which $t' = t$. Lorentz transformations implied that not only the spatial coordinates change when the body moves but that the time coordinate changes too. Minkowski, who introduced the concept of four-dimensional space, noticed that

ct and $\mathbf{r}$ become components of one four-dimensional vector. The difference between squared temporal and spatial components gives an invariant quantity known as interval s squared:

$$s^2 = c^2t^2 - \mathbf{r}^2. \quad (4.5)$$

The four-dimensional space is called, in contrast to the usual three-dimensional space, not Euclidean but pseudo-Euclidean because the invariant here is not the sum but the difference between squares. Note that the difference can be transformed into the sum if instead of t we consider it . As the Minkowski space is isotropic, the magnitude of the four-dimensional interval s , given by formula (4.5) remains unchanged not only under rotations in three spatial planes xy , yz , zx but also in three spatio-temporal planes xt , yt , zt . It is not difficult to see that the last three imaginary rotations correspond to uniform linear motions along the axes x , y , z .

4.7 Energy and momentum in relativistic mechanics

Similarly to this, the isotropic nature of the four-dimensional space implies that energy and momentum of a particle, E/c and $\mathbf{p}$, form a four-dimensional energy-momentum vector, or, as physicists say, the 4-momentum. This 4-momentum squared equals particle's mass squared multiplied by c^2 :

$$m^2c^2 = E^2/c^2 - \mathbf{p}^2 \quad (4.6)$$

As Einstein suggested more than a century ago, in the case of a free body with nonzero mass m this formula reduces to a simple formula

$$E = E_0 + E_K, \quad (4.7)$$

if we introduce, in addition to the concept of kinetic energy E_K , also the concept of rest energy $E_0 = mc^2$ and the concept of the total energy of a free body E .

The velocity of a free particle is associated with its momentum and energy by the relation

$$\mathbf{p} = \mathbf{v}E/c^2. \quad (4.8)$$

Or, in a different form,

$$\mathbf{v} = \mathbf{p}c^2/E. \quad (4.9)$$

Hence as the total energy of a body increases, the increment to its velocity becomes smaller and smaller, and it tends to the limiting value c . This means, however, that Newton's formula eq. (4.1) for kinetic energy becomes inapplicable for $|\mathbf{v}| \approx c$.

The spectacular property of formulas (4.6), (4.8), (4.10) is that they cover not only massive particles but also arbitrarily light particles, and particles whose mass is zero.

4.8 Particle's mass

A different understanding of the mass of a body or particle thus arises. All particles of a given type (e.g. all electrons) are absolutely identical and have the same mass m . For a free particle

$$m^2 = E^2/c^4 - \mathbf{p}^2/c^2. \quad (4.10)$$

4.9 Rest energy

If a free particle is at rest, the rest energy of the particle is given by

$$E(\mathbf{p} = 0) \equiv E_0 = mc^2. \quad (4.11)$$

If the particle moves slowly, then its energy $E \approx E_0$ and formulas (4.7) and (4.8) imply an approximate equality $\mathbf{p} \approx m\mathbf{v}$, which in Newtonian physics takes the form $\mathbf{p} = m\mathbf{v}$.

4.10 Massless photon

Astronomical observations tell us that the photon mass is very small: $m_\gamma < 10^{-51}$ g. Therefore physicists assume that the mass of the photon is zero. Then equation (4.10) implies that the absolute value of photon's momentum is directly proportional to its energy: $|\mathbf{p}| = E/c$ while equation (4.9) implies that its velocity is always c . Free photons cannot be at rest, they always move at the speed of light.

4.11 Masses of electron and proton

The electron mass m_e is now known to the accuracy of 1 in a hundred million, i.e. 10^{-8} and equals approximately $9 \cdot 10^{-28}$ g. The proton mass, known with a comparable accuracy, is approximately two thousand times greater than the electron mass: $m_p \approx 1.7 \cdot 10^{-24}$ g.

Chapter 5

Rotation and Quantization

5.1 The spin and orbital rotation*

Two types of particle rotation are known: spin and orbital.

A particle possesses the spin (the intrinsic rotation) regardless of how it moves through space. It is similar to the Earth's rotation around its axis. The spin rotation is characterized by the spin angular momentum $\mathbf{S}$.

Orbital rotation of a particle resembles Earth's rotation around the Sun. It is characterized by the magnitude of the orbital angular momentum $\mathbf{L}$ which is equal to the vector product of the radius vector $\mathbf{r}$ from the center of rotation to the particle by the particle momentum $\mathbf{p}$:

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}, \quad (5.1)$$

or in vector components:

$$\begin{aligned} L_x &= yp_z - zp_y, \\ L_y &= zp_x - xp_z, \\ L_z &= xp_y - yp_x. \end{aligned} \quad (5.2)$$

It is easy to check that if vector $\mathbf{r}$ is parallel to vector $\mathbf{p}$ then $\mathbf{L} = 0$. In all other configurations $\mathbf{L} \neq 0$. The total

angular momentum equals the sum of the orbital and the spin moments: $\mathbf{J} = \mathbf{L} + \mathbf{S}$.

The angular momentum is called a pseudovector or axial vector because, in contrast to ordinary (polar) vectors $\mathbf{r}$ and $\mathbf{p}$, it does not change sign under mirror reflection. For $\mathbf{L}$ this is immediately obvious from the above definition.

The conservation of the angular momentum of an isolated set of particles is implied by the isotropy of space: empty space has no preferred direction.

5.2 About vectors and tensors

In three-dimensional space, the scalar product of two vectors is a scalar, the vector product of two vectors is an antisymmetric tensor (and at the same time — axial vector) having three components. It is easy to see that a symmetric tensor constructed of two vectors has five (not six) components as it does not contain the scalar: $3 \times 3 = 1 + 3 + 5$.

5.3 The orbital angular momentum in theory of relativity

In relativity theory the three components of (5.1) are a part of the four-dimensional tensor whose other three components have the form:

$$\begin{aligned} N_x &= ctp_x - xE/c, \\ N_y &= ctp_y - yE/c, \\ N_z &= ctp_z - zE/c, \end{aligned} \tag{5.3}$$

or in vector form

$$\mathbf{N} = c t \mathbf{p} - \mathbf{r} E / c. \tag{5.4}$$

If the vectors $\mathbf{r}$ and $\mathbf{p}$ are not parallel to each other, then $\mathbf{N} \neq 0$. If the vectors $\mathbf{r}$ and $\mathbf{p}$ are parallel to each other, then $\mathbf{N} = 0$ and

$$ct\mathbf{p} = \mathbf{r}E/c. \quad (5.5)$$

In the case of translational motion $\mathbf{r} = \mathbf{v}t$ and we derive equations (4.8) and (4.9).

5.4 Identity of particles*

All elementary particles of a given type (e.g. all electrons) are absolutely identical. (Among other things, their masses are exactly the same.) Hence all atoms of a given sort are identical.

5.5 Quantization of $\mathbf{S}$ and $\mathbf{L}$ *

Another remarkable property of nature is that rotation is quantized. The components of $\mathbf{L}$ and $\mathbf{S}$ along some external axis (say z) assume only discrete values with an increment of $\hbar$ where $\hbar$ is the quantum of action introduced into physics in the early twentieth century by Planck.¹

In conditions when quantization is absolutely essential, the motion and interaction of particles is described not by Newtonian mechanics but by quantum mechanics. This is true not only for the theory of elementary particles, theory of atoms and theory of atomic nuclei but also for the theory of molecules and the theory of condensed state.

¹In 1900 Planck introduced the quantity equal to $2\pi\hbar$ into the theory of black body radiation.

According to quantum mechanics the square of the orbital angular momentum $\mathbf{L}$ has to satisfy the equation

$$\mathbf{L}^2 = \hbar^2 l(l+1), \quad (5.6)$$

where $l = 0, 1, 2, \dots$ is the orbital quantum number. To each value of l there correspond $2l + 1$ projections on a certain axis in space: 1, 3, 5 See section 5.2 above.

5.6 More about spin*

A spectacular property of elementary particles is that they have spin — their intrinsic (proper) rotation. I would be inclined to compare them with tiniest tops. However, an ordinary top may not be rotating while the rotation of an elementary particle with nonzero spin cannot stop. The spin differs from the angular momentum of orbital motion in that an absolutely free particle not executing orbital rotation has the spin nevertheless. As will be explained below, the particle spin plays key role in the formation of systems containing many particles.

5.7 Fermions and bosons*

All particles fall into one of two large classes: fermions (with half-integer spin) and bosons (with integer spin). The electron whose spin is $\frac{1}{2}$ is a fermion. The photon whose spin is 1 is a boson. System consisting of two or more identical particles radically differ from each other depending on whether these particles are fermions or bosons.

Not more than one fermion can occupy a given quantum state. Any number of bosons can occupy a given quantum state.

5.8 Elementary quantum state*

Another very important concept in quantum mechanics is that of the elementary quantum state. The elementary quantum state of a free particle is completely characterized by its momentum and orientation of its spin. The elementary quantum state of a bound particle is completely characterized by several quantum numbers (see below). The above-formulated statement on the identity of atoms assumes that atoms occupy identical states. In what follows I will, for the sake of brevity, simply use the term “state” although it would perhaps be wiser to replace the words “elementary quantum state” by some other special term: “state” in quantum mechanics has nearly a dozen different interpretations.

5.9 Bound states*

A bound state of two particles with masses m_1 and m_2 is characterized by the mass of this state $m = m_1 + m_2 - \varepsilon/c^2$ where ε is the binding energy. The quantity ε/c^2 is called the mass defect. In the case of a bound state of an arbitrary number of particles, ε is the total amount of energy required for making every particle free from every other particle of the system.

Chapter 6

Particles as Corpuscles and Waves

6.1 Wave vector*

The constant $\hbar$ plays a most important role not only in rotation of elementary particles but also in their entire behavior. The thing is that unlike ordinary particles (corpuscles) of classical mechanics, elementary particles are some kind of “centaurs”, so to speak, possessing the properties of corpuscles and those of waves at the same time.

The state of a free particle is characterized in quantum mechanics by a certain value of the wave vector $\mathbf{k}$ related to the particle's momentum $\mathbf{p}$ via the de Broglie relation:

$$\mathbf{p} = \hbar \mathbf{k}. \quad (6.1)$$

The quantity $k = |\mathbf{k}|$ is known as the wave number. It is related to the wavelength λ by the formula

$$k = 2\pi/\lambda. \quad (6.2)$$

In the limit in which the wavelength becomes negligibly small, we return to classical mechanics or geometrical optics. (Of course, we mean phenomena in which interference is nonessential.)

A formula similar to (6.1) relates the energy of a particle E with the frequency ω characterizing it (its wave):

$$E = \hbar\omega. \quad (6.3)$$

6.2 The wave function*

As a rule, an introduction into quantum mechanics begins not with the concept of quantum state but with that of the wave function: $\psi(\mathbf{r}, t)$ where $|\psi(\mathbf{r}, t)|^2$ is the probability that the particle has a particular value of $\mathbf{r}$ at the moment of time t . Unfortunately, many people form a very false impression here that in contrast to classical mechanics, in quantum mechanics everything has probabilistic nature. In reality, though, quantum mechanics is in many ways incomparably more rigid than the classical, and we will have a chance to observe it. This inflexibility manifests itself most strikingly in the structure of the Periodic table of chemical elements; to understand it, one only requires the concept of quantum state but does not need to resort to the concept of probability. For this reason, reading sections 6.8–6.10 of this chapter (which deals with wave equations) is not necessary for understanding the structure of the Periodic table.

6.3 Probability amplitude*

The quantum-mechanical probability amplitudes were invariably calculated for more than twenty years by solving wave equations (we shall look at wave equations in more

detail later in this chapter). With this approach, one needs to introduce such concepts as the Hilbert space and the operators acting in it. The majority of textbooks still teach quantum mechanics in this way.

This book attempts not to use these concepts, using instead diagrams invented by Richard Feynman in mid-20th century for calculation of probability amplitudes. These diagrams will be introduced in the chapter devoted to quantum electrodynamics, the QED. They are widely used for calculating probabilities of various processes.

6.4 The role of chance in the decay*

Processes of decay of elementary particles and atomic nuclei and processes of emission of photons by atoms are characterized by the lifetime τ ; of course, lifetimes are different for different processes. The number of particles (or nuclei, or atomic levels) non-decayed over a time τ decreases by a factor of $e \approx 2.7$. (Decay half-life $\tau_{1/2}$, over which the number of non-decayed particles is reduced by half is often used instead of lifetime; $\tau_{1/2} = \tau \ln 2 \approx 0.7\tau$.) Quantum mechanics and relativity theory allow us to predict with a high degree of accuracy the value of τ ; however, we cannot predict in principle when a specific particle will decay. Here chance rules; random phenomena are described by the probability theory.

6.5 The role of chance in the two-slit experiments

The confirmation of wave properties of microscopic particles passing through two slits of the first screen and creating an interference pattern on the second screen standing behind the first played an important role in the evolution of quantum mechanics. The pattern is the sharper, the greater the number of particles passing through the slits. Note that particles can pass through the slits one-by-one; this proves that an individual particle possesses wave properties. However, it is absolutely impossible to predict at what point the particle hits the second screen. Here again chance reigns. Many authors writing about quantum mechanics do not appreciate this behavior and attempt to apply the classical concept of trajectory. Thus a “theoretical interpretation” was produced some time ago of quantum mechanics as a science that claims to describe the coexistence of many different worlds; it continues to cause debates.

6.6 Uncertainty relations*

Heisenberg’s uncertainty relations play a very important role in quantum mechanics:

$$\begin{aligned}\Delta E \Delta t &\geq \hbar/2, \\ \Delta p_x \Delta x &\geq \hbar/2, \\ \Delta p_y \Delta y &\geq \hbar/2, \\ \Delta p_z \Delta z &\geq \hbar/2.\end{aligned}\tag{6.4}$$

.....

These relations establish a connection between the uncertainties with which a particle's energy and the time of its observation, or its momentum or its coordinate can be measured.

6.7 “Correct” and “incorrect” questions*

Quantum mechanics is incomparably more strict than classical mechanics when it answers “correct” questions. But to “incorrect” questions it offers only probabilistic answers. An example of correct question: “What quantum numbers does the particle have in a given state?”. Examples of incorrect questions (incorrect by virtue of the uncertainty relation): “Where a free particle with a specific momentum is located?” or “What is the trajectory of a particle?”. As a result of this, such very important concept of classical physics as location in the world (in four-dimensional spacetime) becomes probabilistic (“smeared”).

6.8 Schrödinger equation

In the nonrelativistic limit the wave function satisfies the Schrödinger wave equation:

$$i\frac{\partial\psi}{\partial t} = \hat{H}\psi, \quad (6.5)$$

where $\hat{H}$ is the Hamiltonian operator known as the Hamiltonian; it equals the sum of the kinetic and potential energy operators. For a charged particle in electromagnetic field $\phi, \mathbf{A}$ it has the form:

$$\hat{H} = \frac{1}{2m}(\hat{\mathbf{p}} - e\hat{\mathbf{A}})^2 - e\phi, \quad (6.6)$$

where $\hat{\mathbf{p}} = -i/\partial\mathbf{r}$ is the momentum operator. The wave function ψ is the set (linear superposition) of all possible quantum states of a given physical system.

6.9 The Klein–Fock–Gordon equation

The equation for a relativistic spinless particle was found in 1926 by Klein, Fock and Gordon. For a free particle it has the form

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} + m^2\right)\psi = 0. \quad (6.7)$$

In four-dimensional notation, it can be written as

$$\left(\frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} + m^2\right)\psi = 0. \quad (6.8)$$

Here x^μ is a contravariant four-dimensional vector and x_μ is a covariant four-dimensional vector. Repeated (“dummy”) indices imply summation: $x_\mu = g_{\mu\nu}x^\nu$.

The relativistically invariant wave function of a spinless charged particle in electromagnetic field $A_\mu = \phi, \mathbf{A}$ satisfies the equation

$$\left(\left(\frac{\partial\psi}{\partial x^\mu} - ieA_\mu\right)\left(\frac{\partial\psi}{\partial x_\mu} - ieA^\mu\right) + m^2\right)\psi = 0. \quad (6.9)$$