

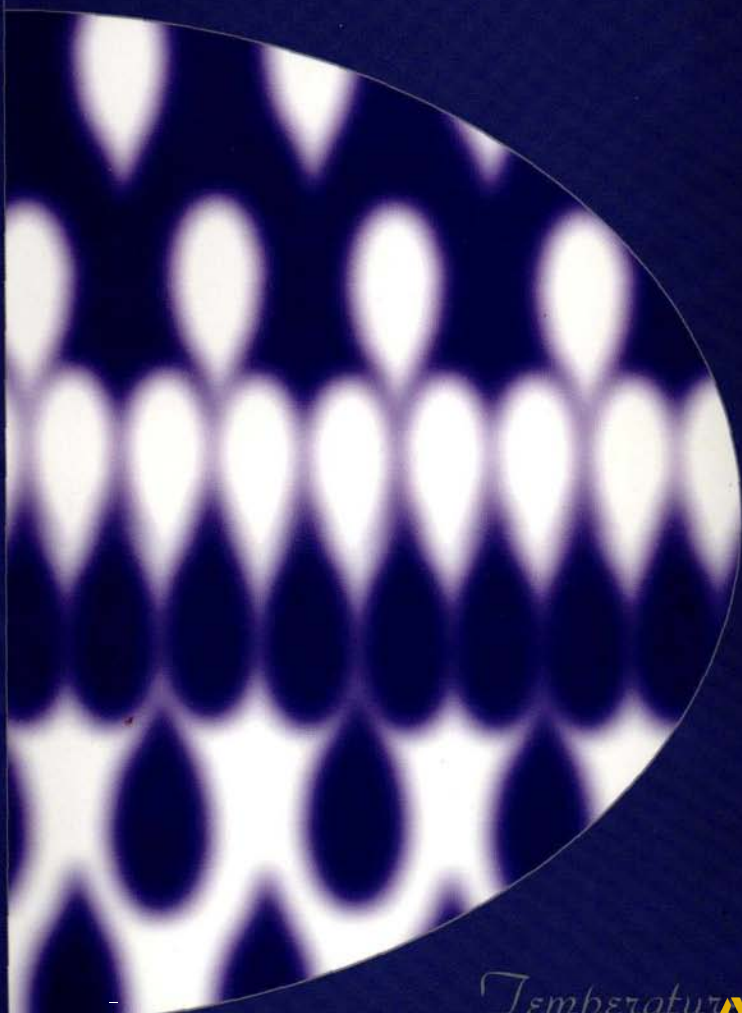
Lecture Notes on

Phase Transformations in Nuclear Matter

Jorge A. López

Claudio O. Dorso

Density



Temperature **AVA**

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**Phase Transformations
in Nuclear Matter**

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Phase Transformations in Nuclear Matter

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LECTURES ON PHASE TRANSFORMATIONS IN NUCLEAR MATTER

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This book is printed on acid and chlorine free paper.

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En este trabajo intervinieron:

Mi familia: Rosa Elena, Maribel y Oscar,
quienes me dieron su tiempo y su amor.

Mis maestros: Roberto Díaz Molina, Roberto Armijo,
Alan Dean, y Philip Siemens,
quienes me guiaron y alentaron.

Y la naturaleza,
*que por ser tan lineal,
nos puso en el camino correcto
para empezar a entender su complejidad.*

Jorge

A mi familia:

Alicia, Ezequiel, Federico, Sabrina,

Elsa, José, Nelly y Beto.

A quienes contribuyeron en mi formación:

E. S. Hernández y J. Randrup.

Claudio

Preface

Nuclear matter is unique in several ways. The fact that, from the light to the heavy, all nuclei have an almost constant density and binding energy shows the strength of the nuclear force as well as its short range. Nucleons in small nuclei feel roughly the same pull from neighboring nucleons as in large nuclei. Thus the famous way of counting nucleons “*One, Two, Three, Four, Infinity*”; after a few nucleons all nuclei are pretty much alike in terms of density and binding energy.

Constant density implies a constant specific volume per nucleon. What keeps the nucleons at that fixed distance from one another? Naively speaking, it has to be a force produced by a strong potential with a narrow and pointed minimum at the right place. And to have stable nuclei, that potential has to have a repulsive core as well as an attractive short tail. Much like a van der Waals gas.

This book deals with some of the exciting possibilities that a van der Waals nuclear liquid has in terms of thermodynamics and phase transformations. Such nuclear systems can be found in nuclei as well as in astrophysical objects. Its behavior in terms of its response to external forces determines, for instance, the rate of oscillations in monopole excitations, the amount of energy stored as heat, and the kinetic energy released through evaporation of light particles during collisions.

The study of phase transformations of nuclear matter was promoted in the decade of the 80's when it began to be experimentally possible to achieve energetic collisions between heavy nuclei with beam energies in the tens of *MeV*s per nucleon. Starting with an initially idealized picture, the reactions were studied as a perfect laboratory example of a thermodynamic system.

After these initial simple models explained some of the existing inclusive data, the experiments began to provide detailed information that forced a change of the physical picture of the process. It is now believed that heavy ion reactions leading to fragmentation, although achieving a high degree of thermalization, are dominated mainly by finite-size and collision-induced correlations of nucleon density and momentum.

As of now, there is no complete theory to study such complex processes (e.g. non-equilibrium critical phenomena in evolving small quantum systems). Our best hope to understand heavy ion reactions, then, is to start with a simple model and gradually add those ingredients required to reach higher degrees of complexity. This work studies the nuclear fragmentation problem starting from a free nucleon gas, adding next a phenomenological nucleon interaction to study possible phase transitions in infinite and finite systems, and to compare to experimental data.

Restating this plan in further detail, chapter 1 introduces a simple free nucleon gas to develop a baseline model to obtain computational methodology, as well as basic medium information and phenomena, such as temperature scales and nucleon evaporation. Adding a phenomenological binding energy, chapter 2 focuses on the resulting nuclear equation of state and its thermodynamic properties. The liquid-gas phase transition is investigated in more detail in chapter 3, where isothermal and isentropic nucleation processes are explored along with spinodal decomposition and other related topics. The fact that the nucleus is a small system is introduced in chapter 4 along with some of the existing techniques for its study. Finally, and since both authors are theorists, the connection to experimental data is timidly presented in chapter 5.

For completeness, the book closes with a brief summary of commonly used computational techniques and some useful programs. [Some of these programs and other interesting ones can be downloaded from the author's web page at www.utep.edu/physics or by anonymous ftp from [zapata.utep.edu](ftp://zapata.utep.edu).

On the technical aspects of the production of this book, the authors kindly acknowledge help from a variety of sources, including, but not limited to Azael Avalos, Pablo Balenzuela, Mario Borunda, Ariel Chernomoretz, Christian Escudero, Martha Navarro, and Rocío Olave. Jorge López also thanks the warm hospitality of the Universidad de Buenos Aires where part of the manuscript was prepared.

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March, 2000

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March, 2000

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Chapter 1

The Free Nucleon Gas

1.1 Introduction

In its simplest model, a nucleus can be represented as a collection of neutrons and protons in a potential well [Segrè (1982)]. When the number of nucleons is not too small, the volume is restrictive enough to warrant the use of statistical mechanics. In this first approximation, the nucleus is treated as an aggregate of nucleons interacting only through the exclusion principle. This simple model, which will allow us to set the formulae, nomenclature, mass and energy scales to be used throughout the book, it also gives a good representation of the nucleus at low energies, as shown by the Fermi-type energy distributions of nucleons extracted from low energy electron-nucleus scattering [Bohr and Mottelson (1973)].

1.2 The Free Nucleon Gas

For a set of N indistinguishable nucleons in a volume V , at a temperature T , and in a potential well with energy states $\epsilon_1, \dots, \epsilon_k$, and occupational numbers $n_1, \dots, n_k, \dots$, the canonical partition function is given by [McQuarrie (1973)]

$$*Q(N, V, T) = \sum_{i,j,k,\dots} e^{-\beta(\epsilon_i + \epsilon_j + \dots)} = \sum_{\{n_k\}}^* e^{-\beta \sum_i \epsilon_i n_i},$$

where $\beta = 1/T$ and the $\sum^*$ indicates that the sum must be performed with the awkward restriction of keeping the total number of particles $N = \sum_k n_k$

fixed. [In nuclear physics it is customary to express the temperature T directly in energy units of MeV , formally this is equivalent to $k_B T$ where T is the temperature in Kelvin.]

To proceed, and since the sum $\sum^*$ is difficult to complete, it is convenient switch to the grand canonical ensemble where V , T , and the chemical potential μ are the independent variables. The grand canonical partition function is given by

$$\Xi(V, T, \mu) = \sum_{N=0}^{\infty} e^{-\beta\mu N} Q(N, V, T) = \sum_{N=0}^{\infty} \sum_{\{n_k\}}^* \prod \lambda e^{(-\beta\epsilon_k)n_k},$$

where $\lambda = e^{\beta\mu}$. Now, since the nucleons are fermions, $n_k = 0, 1$ and the partition function reduces to $\Xi_{FD} = \prod_k (1 + \lambda e^{-\beta\epsilon_k})$.

With this, one can now extract many thermodynamic variables of interest. The average number of particles, for instance, is given by

$$N = \sum_k \bar{n}_k = T \left(\frac{\partial \ln \Xi}{\partial \mu} \right)_{V, T} = \sum_k \frac{\lambda e^{-\beta\epsilon_k}}{1 + \lambda e^{-\beta\epsilon_k}}, \quad (1.1)$$

i.e. the average occupation number is $\bar{n}_k = \lambda e^{-\beta\epsilon_k} / (1 + \lambda e^{-\beta\epsilon_k})$. Likewise, one can obtain the average total energy, pressure and entropy:

$$E_F = \sum_k \bar{n}_k \epsilon_k = \sum_k \lambda \epsilon_k e^{-\beta\epsilon_k} / (1 + \lambda e^{-\beta\epsilon_k}), \quad (1.2)$$

$$pV = T \ln \Xi(V, T, \mu) = T \sum_k \ln(1 + \lambda e^{-\beta\epsilon_k}), \quad (1.3)$$

$$S = \left[\frac{\partial pV}{\partial T} \right]_{V, \mu} = \sum_k \left[\ln(1 + \lambda e^{-\beta\epsilon_k}) - \frac{(\mu - \epsilon_k) e^{\beta(\mu - \epsilon_k)}}{T(1 + \lambda e^{-\beta\epsilon_k})} \right]. \quad (1.4)$$

In the nuclear case the energies ϵ_k 's are almost continuously distributed (cf. problem 1.4), and one can go from $\sum_k$ to $\int \omega(\epsilon) d\epsilon$, where $\omega(\epsilon) d\epsilon$ is the number of states with energy between ϵ and $\epsilon + d\epsilon$. Using $\omega(\epsilon)$ from problem 1.1 immediately yields

$$N = 2g\pi \left(\frac{2m}{h^2} \right)^{3/2} V \int_0^{\infty} \frac{\lambda \epsilon^{1/2} e^{-\beta\epsilon} d\epsilon}{1 + \lambda e^{-\beta\epsilon}}, \quad (1.5)$$

$$E_F = 2g\pi \left(\frac{2m}{h^2} \right)^{3/2} V \int_0^{\infty} \frac{\lambda \epsilon^{3/2} e^{-\beta\epsilon} d\epsilon}{1 + \lambda e^{-\beta\epsilon}}, \quad (1.6)$$

and

$$\begin{aligned}
 p &= 2g\pi T \left(\frac{2m}{h^2}\right)^{3/2} \int_0^\infty \epsilon^{1/2} \ln(1 + \lambda e^{-\beta\epsilon}) d\epsilon \\
 &= \frac{4}{3} g\pi \left(\frac{2m}{h^2}\right)^{3/2} \int_0^\infty \frac{\epsilon^{3/2} d\epsilon}{(1 + \lambda e^{\beta\epsilon})} = \frac{2}{3V} E_F, \quad (1.7)
 \end{aligned}$$

where the $\ln$ integral in p was reduced using integration by parts. For the nuclear case $g = 4$ to take into account the spin and isospin degrees of freedom. In most cases, the evaluation of expressions such as (1.5)-(1.7) needs the use of numerical integration. In some limits, however, it is possible to approximate the Fermi integrals, this is done for the nuclear case in the following section.

Problem 1.1 The Energy Level Density

(A) Consider a three dimensional well with energy levels $\epsilon = g\hbar^2(n_x^2 + n_y^2 + n_z^2)/8mV^{2/3}$, where $\{n_i\}$ and g are the quantum numbers and degeneracy, respectively. Using geometrical arguments in n_i -space, show that the number of states available to a particle with energy $\leq \epsilon$ is

$$\Phi(\epsilon) = \frac{\pi}{6} \left[\frac{8mV^{2/3}\epsilon}{h^2} \right]^{3/2}.$$

(B) Likewise, show that the number of states in a shell of energy ϵ and thickness $d\epsilon$ is

$$\omega(\epsilon)d\epsilon = 2g\pi \left(\frac{2m}{h^2}\right)^{3/2} V \epsilon^{1/2} d\epsilon. \quad (1.8)$$

1.3 Strongly Degenerate Nucleon Gas

For excitations achieved in intermediate-energy reactions, one can argue that the system maintains high level occupancy and is never weakly degenerate (cf. problem 1.9). In this limit where the density is high (near saturation density n_0) and or the temperature low ($T \ll \epsilon_F$, cf. problem 1.2), most of the states are occupied and $\epsilon \approx \mu$. Expressions (1.5)-(1.7) can be simplified through an expansion in terms of $(\epsilon - \mu)$.

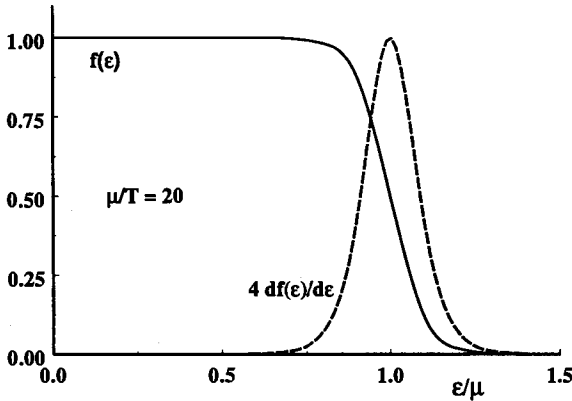


Fig. 1.1 Behavior of $f(\epsilon)$ and $df/d\epsilon$ as a function of ϵ/μ

Expression (1.1) shows that $\bar{n}_k = 1/(1 + e^{\beta(\epsilon_k - \mu)})$, is the probability that state k is occupied. For continuous energy distributions this probability is denoted by $f(\epsilon) = 1/(1 + \lambda e^{\beta\epsilon})$. Figure 1.1 shows the energy dependence of $f(\epsilon)$ as a function of ϵ/μ , clearly $f(\epsilon) \approx 1$ for $\epsilon \leq \mu$ and $f(\epsilon) = 0$ for $\epsilon \geq \mu$. That is, all states with energy $\epsilon \leq \mu$ are occupied and all other are empty. This helps to obtain simple analytic expressions for the zero temperature case.

Problem 1.2 The $T = 0$ Case and the Fermi Energy

(A) Integrate (1.5) to show that in the zero-temperature limit N is given by $N = (4g\pi/3)(2m/\hbar^2)^{3/2}V\mu_0^{3/2}$, where μ_0 is the $T = 0$ value of the chemical potential, known as the Fermi energy, ϵ_F . Likewise show that the $T = 0$ value of the total energy of the free nucleon gas is given by $E_0 = 3N\mu_0/5$, and the pressure by $p_0 = 2N\mu_0/(5V)$.

(B) Extract the Fermi energy ϵ_F in terms of N , and evaluate it for the nuclear case where $N/V \approx 0.15 \text{ fm}^{-3}$ and $m \approx 1000 \text{ MeV}/c^2$. Estimate E_0 and p_0 for the nuclear case too.

For low temperature ($T \ll \epsilon_F$) analytic expressions can be obtained through the use of approximations, here the case of the pressure is worked as an illustration. The integral of expression (1.7) can be evaluated by parts to

obtain

$$I = \int_0^\infty \frac{\epsilon^{3/2} d\epsilon}{(1 + \lambda e^{\beta\epsilon})} = \frac{2\beta}{5} \int_0^\infty \frac{\epsilon^{5/2} e^{\beta(\epsilon-\mu)} d\epsilon}{(1 + e^{\beta(\epsilon-\mu)})^2}.$$

This last integral cannot be evaluated analytically, but, as seen in figure 1.1, the factor $df(\epsilon)/d\epsilon = e^{\beta(\epsilon-\mu)}/(1 + e^{\beta(\epsilon-\mu)})^2$ is strongly peaked around $\epsilon \approx \mu$ for low values of T , and one can thus use the expansion $\epsilon^{5/2} = \mu^{5/2} + (\epsilon - \mu)(\partial\epsilon^{5/2}/\partial\epsilon)_{\epsilon=\mu} + (1/2)(\epsilon - \mu)^2(\partial^2\epsilon^{5/2}/\partial\epsilon^2)_{\epsilon=\mu} + \dots$ to obtain

$$\begin{aligned} I &= \frac{2\beta\mu^{5/2}}{5} \int_0^\infty \frac{e^{\beta(\epsilon-\mu)} d\epsilon}{(1 + e^{\beta(\epsilon-\mu)})^2} + \beta\mu^{3/2} \int_0^\infty \frac{(\epsilon - \mu)e^{\beta(\epsilon-\mu)} d\epsilon}{(1 + e^{\beta(\epsilon-\mu)})^2} \\ &+ \frac{3\beta\mu^{1/2}}{4} \int_0^\infty \frac{(\epsilon - \mu)^2 e^{\beta(\epsilon-\mu)} d\epsilon}{(1 + e^{\beta(\epsilon-\mu)})^2} + \dots \end{aligned}$$

The first integral equals simply $2\beta\mu^{5/2}/5$, and to approximate the rest of the integrals one extends the lower limit of integration to $-\infty$. All those integrals with odd powers of $(\epsilon - \mu)$ have odd integrands and are thus equal to zero. The remaining integrals are given by $I_i = \int_{-\infty}^\infty t^i e^t dt / (1 + e^t)^2 = 2i(i-1)!(1 - 2^{1-i})\zeta(i)$, for i integer and even, and where $\zeta(i)$ is the Riemann zeta function. The degenerate limit of the pressure is then given by

$$p = \frac{8g\pi}{15} \left(\frac{2m}{h^2} \right)^{3/2} [\mu^{5/2} + 5\pi^2 T^2 \mu^{1/2} + \dots]. \quad (1.9)$$

A similar derivation for N gives

$$N = \frac{8g\pi}{3} \left(\frac{2m}{h^2} \right)^{3/2} V [\mu^{3/2} + \frac{\pi^2}{8} T^2 \mu^{-1/2} + \dots]. \quad (1.10)$$

The corresponding expression for E_F can be obtained from equation (1.7), and (1.9). The entropy can be obtained from these previous expressions and the fundamental Gibbs-Duhem equation is $S = E_F/T - N\mu/T + pV/T$.

Problem 1.3 Strongly Degenerate Expression for N

Derive expression (1.10) starting from (1.5). Use it to sketch the behavior of the number density $n = N/V$ for the range $1 \text{ MeV} < T < 10 \text{ MeV}$ and $20 \text{ MeV} < \mu < 40 \text{ MeV}$.

So far μ and T have been used as the independent variables, but sometimes it is more convenient to use the density and temperature instead. This can

be achieved by inverting expression (1.10) to obtain the chemical potential as a function of T and ϵ_F (see *eg.* [McQuarrie (1973)]):

$$\mu = \epsilon_F \left[1 - \frac{\pi^2}{12} \left(\frac{T}{\epsilon_F} \right)^2 - \frac{\pi^4}{180} \left(\frac{T}{\epsilon_F} \right)^4 + \dots \right]. \quad (1.11)$$

This can be used to obtain

$$E_F = \frac{3}{5} N \epsilon_F \left[1 + \frac{5\pi^2}{12} \left(\frac{T}{\epsilon_F} \right)^2 - \frac{\pi^4}{16} \left(\frac{T}{\epsilon_F} \right)^4 \dots \right]. \quad (1.12)$$

The thermal energy $\epsilon_T(n, T) = (E_F(n, T) - E_F(n, 0))/N$ as obtained with approximation (1.12) is compared in figure 1.2 to a numerical evaluation and a quadratic fit that will be introduced in the next sections.

The heat capacity can also be calculated as

$$C_V = \left(\frac{\partial E_F}{\partial T} \right)_V = \frac{\pi^2 N k^2 T}{2\epsilon_F} + \dots. \quad (1.13)$$

These expressions can be evaluated using the result of problem 1.2, namely, $\epsilon_F = (h^2/2m)(3N/8g\pi V)^{2/3}$.

1.3.1 Energy Level Density

In the present free nucleon gas approximation, the only interaction between nucleons is that of the exclusion principle. This can be highly restrictive for the particle dynamics at low temperatures. To quantify the number of available energy states, the previously derived strongly degenerate expressions will be used to obtain a crude estimate of the density of energy states.

From the statistical and thermodynamic definitions of entropy,

$$S(E_F(T)) = \log \left[\frac{\omega(E_F(T))}{\omega(E_F(0))} \right], \quad S(E_F(T)) = \int_0^T dE_F(T)/T,$$

and using equation (1.12), one obtains $S = N\pi^2 T/2\epsilon_F$. Now, defining the level density parameter as $a = N\pi^2/4\epsilon_F$ and since $E_T(T) = N\pi^2 T^2/4\epsilon_F = aT^2$, the entropy can be written as $S = 2\sqrt{aE_T}$. With this the energy level density is

$$\omega(E_F(T)) = \omega(E_F(0)) e^{2\sqrt{aE_T(T)}}. \quad (1.14)$$

It is worth noticing that in this derivation the entropy does not contain contributions from volume and particle number changes, i.e. it corresponds to the case of fixed volume and N .

The ratio $\omega(E_F(T))/\omega(E_F(0))$ can be taken as a measure of the opening of the phase-space as a function of E_T . For instance, as shown in problem 1.4, the number of available states for 100 nucleons increases by a factor of a million at energies as E_T goes from zero to 4 MeV. The restrictions imposed by Pauli's principle should be of no concern in the nuclear case.

Problem 1.4 Nuclear Level Density

(A) Fill in the details in the derivation of expression (1.14).

(B) Evaluate the level density parameter a for the nuclear case and show that $a \sim N/8$ at n_0 .

(C) Sketch the ratio $\omega(E_F(T))/\omega(E_F(0))$ for $E_F = 7$ MeV and the range $10 < N < 240$. [You can compare your gross estimate to the experimental data of figure 11.29 of [Segrè (1982)] where $\omega(E_F(0))$ and a have been adjusted to fit the data.]

1.4 Numerical Evaluations

For the general case, expressions (1.5)-(1.7) must be evaluated numerically. Letting $g = 4$ and $\eta = \mu/T$, expressions (1.5) and (1.7) can be written as

$$n = \frac{N}{V} = \frac{1}{\pi^2} \left(\frac{2mc^2}{(\hbar c)^2} \right)^{3/2} T^{3/2} \int_0^\infty \frac{t^{1/2} dt}{1 + e^{t-\eta}}$$

$$p = \frac{2}{3\pi^2} \left(\frac{2mc^2}{(\hbar c)^2} \right)^{3/2} T^{5/2} \int_0^\infty \frac{t^{3/2} dt}{1 + e^{t-\eta}}.$$

The Fermi integral $\int_0^\infty t^i dt/(1 + e^{t-\eta})$ and can be evaluated using any standard package, such as the *FERDR* subroutine *C323* of the *CERN* library (conveniently reproduced in appendix B.2). Furthermore, to eliminate the dependence on the number of particles, the energy per particle can be evaluated from

$$\varepsilon_F = E_F/N = \frac{3p}{2n} = T \frac{\int_0^\infty t^{3/2} dt/(1 + e^{t-\eta})}{\int_0^\infty t^{1/2} dt/(1 + e^{t-\eta})}.$$

The numerical evaluation of the thermal energy of the free nucleon gas, $\varepsilon_T(n, T) = (E_F(n, T) - E_F(n, 0))/N$, is compared in figure 1.2 to the

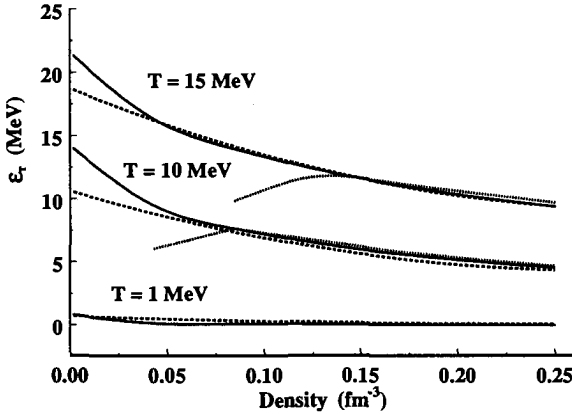


Fig. 1.2 Behavior of a numerical integration (solid), quadratic fit (dashed) and the strongly degenerate expansion (dotted) of $\epsilon_T(n, T) = (E_F(n, T) - E_F(n, 0))/N$ as a function of the density n and for three different temperatures.

strongly degenerate expansion of section 1.3, and to a fit to be introduced in the next section.

Problem 1.5 Numerical Evaluation of n , p , ϵ and s

Use code `Fermigas.for` from appendix B.1 and subroutine `Ferdr.for` from appendix B.2 to evaluate numerically n , p , ϵ and the entropy per particle, $s = S/N$, for the ranges $1 \text{ MeV} < T < 10 \text{ MeV}$ and $20 \text{ MeV} < \mu < 40 \text{ MeV}$. Compare your number density results to those of problem 1.3.

1.5 Useful Fits

Although a numerical integration of the thermodynamic expressions of the free nucleon gas is more generically applicable, it is possible to obtain fits to these expressions which are valid within a specific range of the independent variables. For instance, the temperature part of the energy per particle, namely $\epsilon_T(n, T) = (E_F(n, T) - E_F(n, 0))/N$ can be fitted by a doubly

Table 1.1 Coefficients ε_{ij}^T and ε_{ij}^S

Coefficient	Value	Coefficient	Value
ε_{01}^T	0.693	ε_{02}^T	0.037
ε_{11}^T	-5.420	ε_{12}^T	0.082
ε_{21}^T	11.447	ε_{22}^T	-0.312
ε_{01}^S	-0.0984	ε_{02}^S	0.845
ε_{11}^S	-2.481	ε_{12}^S	20.607
ε_{21}^S	-1.697	ε_{22}^S	17.151

quadratic polynomial of the form

$$\varepsilon_T(n, T) = \sum_{i=0}^2 \varepsilon_{Ti}(T) n^i,$$

with the temperature-dependent coefficients given by $\varepsilon_{Ti}(T) = \sum_{j=1}^2 \varepsilon_{ij}^T T^j$.

A similar fit can be obtained if ε is needed in terms of the entropy: $\varepsilon_S(n, S) = \sum_{i=0}^2 \varepsilon_{Si}(S) n^i$ with the entropy-dependent coefficients given by $\varepsilon_{Si}(S) = \sum_{j=1}^2 \varepsilon_{ij}^S S^j$. Table 1.1 lists the values of the coefficients $\{\varepsilon_{ij}^T\}$ and $\{\varepsilon_{ij}^S\}$ that best reproduce ε_T and ε_S , respectively for the ranges of interest in the nuclear case: $0 < T < 20 \text{ MeV}$, $0 < S < 3$, and $0 < n < 0.2 \text{ fm}^{-3}$. [Notice that in the present units S is unitless, i.e. it is measured in units of Boltzmann's constant.] Figure 1.2 compares the temperature dependent fit of $\varepsilon_T(n, T)$ to the degenerate expression and the numerical evaluation presented in sections 1.3 and 1.4, respectively.

1.5.1 Caloric Curve

The fits obtained for the thermal energy, ε_T , can help us study the so-called *caloric* curve. This the existing relationship between the temperature of the free nucleon gas and its thermal energy.

Since the thermal energy, $\varepsilon_T(n, T)$, depends explicitly on T through a simple doubly quadratic polynomial, the caloric curve can be obtained by inverting this expression for T . This procedure yields

$$T(n, \varepsilon_T) = -T_B(n) + \frac{\sqrt{T_B^2(n) + 4 T_A(n) \varepsilon_T}}{2 T_A(n)},$$

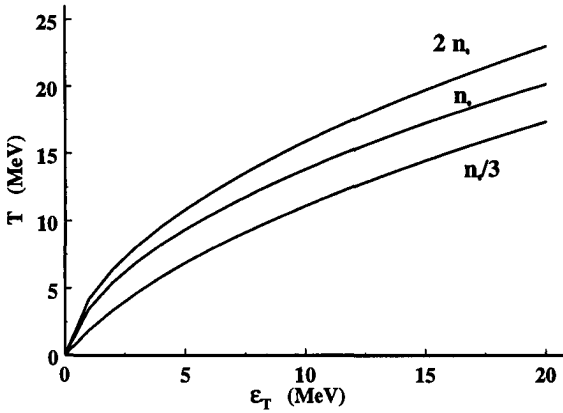


Fig. 1.3 Caloric curve for the free nucleon gas at constant densities

where $T_A(n) = \sum_{i=0}^2 \epsilon_{i2}^T n^i$ and $T_B(n) = \sum_{i=0}^2 \epsilon_{i1}^T n^i$. Figure 1.3 shows the $T - \epsilon_T$ relationship at three values of the density going from one third to twice saturation density.

For real gases, the caloric curve (at constant pressure) has the peculiarity of being piecewise continuous at phase changes. As the present model does not contain nucleon-nucleon interactions, its caloric curve does not present these features. For a comparison to a caloric curve extracted from experimental data see figure 4.10 in section 4.5 or figure 5.4 in section 5.5.

1.6 Nucleon Evaporation

Although, due to the lack of particle-particle interactions, the free nucleon gas does not present phases, it can “*evaporate*” particles. Evaporation in this context refers to the separation of particles from the main body of the gas. In this section, as an application of the free nucleon gas model, two alternative nucleon radiation techniques will be studied: thermionic emission and Weisskopf evaporation. For a comparison of these methods, see [Friedman and Lynch (1983)].

1.6.1 Thermionic Emission

Consider a gas of free nucleons contained in a square well potential of height W , with the property $W > \epsilon_F$. The momentum distribution is given by $f(\epsilon)$ (cf. section 1.3) expressed in terms of the momentum $\vec{p}$: $f(\vec{p}) = 1/(1 + e^{(\vec{p}^2/2m - \mu)/T})$. If the nucleons are contained in a region of space separated from the vacuum by a surface, those particles in the high-momentum end of the distribution will be able to escape into the vacuum.

Placing the surface perpendicularly to the z -direction, particles with $p_z^2/2m > W$ will escape in a unit time provided they are at a distance $dz = v_z$ from the surface, where v_z is the velocity perpendicular to the surface. The number of particles escaping per unit time per unit surface is then given by

$$R = \int_{p_z = \sqrt{2mW}}^{\infty} \int_{p_{xy}} d^3p \, n_p \, v_z \, f(\vec{p}) ,$$

where p_{xy} is the component of the momentum transverse to the z direction, $v_z = p_z/m$, and n_p is the number of quantum states per unit volume, i.e. $n_p = (gV/h^3)/V = g/h^3$. Using $d^3p = dp_z dp_{xy} d\theta_{xy}$

$$\begin{aligned} R &= \int_{p_z = \sqrt{2mW}}^{\infty} \int_0^{\infty} \int_0^{2\pi} \frac{g}{mh^3} \frac{d\theta \, p_{xy} dp_{xy} \, p_z dp_z}{(1 + e^{((\vec{p}_x^2 + \vec{p}_y^2)/2m - \mu)/T})} , \\ &= \frac{2g\pi mT}{h^3} \int_{\epsilon_z = W}^{\infty} d\epsilon_z \ln [1 + e^{(\mu - \epsilon_z)/T}] , \end{aligned}$$

where a change of variable to $\epsilon_z = \vec{p}_z^2/2m$ has been introduced. Now, for low temperatures ($T \ll W$), and since $W > \mu$ the exponential term will be small and thus

$$R \approx \frac{2g\pi mT}{h^3} \int_W^{\infty} d\epsilon_z e^{(\mu - \epsilon_z)/T} = \frac{2g\pi mT^2}{h^3} e^{(\mu - W)/T} . \quad (1.15)$$

Since μ is a function of the temperature and density, R depends effectively on these variables too.

Problem 1.6 Thermionic Emission

Use expression (1.15) to calculate the number of nucleons that radiate away from a $T = 3 \text{ MeV}$, $N = 100$ nucleus in a time interval of 10^{-21} sec . Take $\mu - W$ equal to the separation energy 8 MeV , and estimate the nuclear surface taking the nucleus as a sphere of radius $r = r_0 N^{1/3}$ with $r_0 = 1.2 \text{ fm}$.

The simple free nucleon gas used here does not distinguish between neutrons and protons. In a more realistic model (see *e.g.* [López and Randrup (1994)]), escaping protons would feel a Coulomb repulsion of the form $W_{\text{protons}} = e^2(Z-1)/r$ (where Ze and r are the nuclear charge and radius), whereas for the neutrons, $W_{\text{neutrons}} = 0$. It is worth mentioning that the product $R \times \text{Surface} \times \text{Time}$ is a dN/dt , and as such it should be integrated when calculating the emission over long periods of time as N and T might change.

1.6.2 Weisskopf Evaporation

An alternative, but equivalent way of calculating the emission of nucleons from an excited free nucleon gas comes from the statistical theory of nuclei [Weisskopf (1937)]. According to this theory, the rate of decay (*i.e.* decays per unit time) of a nucleus “ C ” into a nucleon “ b ” and a daughter nucleus “ B ” is given by

$$\frac{dN^2}{dK dt} = \frac{gm}{\pi^2 \hbar^3} \sigma_{b+B}^c K \frac{\omega_B(E_B)}{\omega_c(E_c)}, \quad (1.16)$$

where m , K , and g refer to the mass, kinetic energy and degeneracy of the emitted nucleon, $\omega_B(E_B)$ and $\omega_c(E_c)$ are the level densities of the daughter and mother nuclei at *thermal* energies E_B and E_c , respectively, and σ_{b+B}^c is the $b + B \rightarrow c$ reaction cross section.

To integrate over all values of K , a level density appropriate for the case when N is not held constant will be used. Keeping the volume constant, and proceeding as in section 1.3.1, the entropy and level density are given by

$$S = \int_0^T dE/T - \int_0^N (\mu/T) dN = 2\sqrt{aE_T} - (\mu/T)N, \\ \omega(E) = \omega(0)e^{2\sqrt{aE_T} - (\mu/T)N}.$$

Now since $E_c = E_B + K + W$, and $N_c = N_B + 1$, the ratio in (1.16) can be expressed as

$$\frac{\omega_B(E_B)}{\omega_c(E_c)} = e^{2\sqrt{aE_c}(\sqrt{1-(K+W)/E_c-1}) + \mu/T} \approx e^{-(K+W)/T + \mu/T},$$

where $\sqrt{1-(K+W)/E_c}$ has been expanded to first order, and $\sqrt{a/E_c}$ has been replaced by T as in section 1.3.1. The nucleon emission per unit

time is thus

$$\frac{dN}{dt} = \frac{g m \sigma_{b+B}^c}{\pi^2 \hbar^3} \int_0^{E_c+W} K e^{-(K+W)/T+\mu/T} dK \approx \frac{g m \sigma_{b+B}^c}{\pi^2 \hbar^3} T^2 e^{(\mu-W)/T},$$

where, taking advantage of the fast decreasing exponential, the upper limit of integration has been replaced by ∞ . Taking the emitting surface as $4\sigma_{b+B}^c$, the number of particles escaping per unit time per unit surface is given by

$$R = \frac{2 g \pi m}{h^3} T^2 e^{(\mu-W)/T}, \quad (1.17)$$

in agreement with the thermionic emission expression (1.15). Expression (1.17) was derived keeping the volume fixed, reference [Friedman and Lynch (1983)] extends the present approach to the case of constant density.

1.7 Some Conclusions

The free nucleon gas is the simplest model of nuclear matter. In this case, statistical mechanics can be applied due to the relatively large number of nucleons for the restrictively small volume. Using only the exclusion principle, the model represents the nucleus as a collection of neutrons and protons in a potential well with a density of energy levels not too different from the real one. Although computationally demanding, the model can be approximated by the use of numerical expansions, integrations and fits. The nuclear scales of interest that result from this model are: $0 < T < 20 \text{ MeV}$, $0 < S < 3$, $0 < n < 0.2 \text{ fm}^{-3}$, $20 \text{ MeV} < \mu < 40 \text{ MeV}$, and $0 < \varepsilon_T < 40 \text{ MeV}$ with a value of N in the hundreds of particles and $n_0 = 0.15 \text{ fm}^{-3}$. In terms of dynamical phenomena, this simple model predicts the possibility of nucleon evaporation.

1.8 Additional Problems

Problem 1.7 Obtaining N , E_F , and p

Obtain N , E_F and p (expressions (1.5)-(1.7)) starting from equations (1.1)-(1.3) and using expression (1.8).

Problem 1.8 Entropy

Show that $S = E_F/T - N\mu/T + pV/T$ starting from expressions (1.4) and (1.8).

Problem 1.9 Weak Degeneracy

Weak degeneracy sets in when the de Broglie wavelength, Λ , of the nucleons is roughly of the size of the nucleon's volume, i.e. $\Lambda^3 \approx V/N$, where $\Lambda = \sqrt{h^2/(2\pi mT)}$. For weak degeneracy, the ratio $N\Lambda^3/V$ would have to be approximately close to unity. Sketch the behavior of $N\Lambda^3/V$ for the nuclear case using $N/V = 0.15 \text{ fm}^{-3}$, $m = 938 \text{ MeV}/c^2$, and T ranging from 0 to 20 MeV's. What is a "safe" temperature limit for weak degeneracy?

Problem 1.10 Strongly Degenerate Expression for μ and E

Derive expressions (1.11)-(1.13). Hint: see section 10.2 in [McQuarrie (1973)].

Problem 1.11 Helmholtz Free Energy

Show that the strongly degenerate expression for the Helmholtz free energy is given by $A = (3N\epsilon_F/5)[1 - (5\pi^2/12)(T/\epsilon_F)^2 + (\pi^4/48)(T/\epsilon_F)^4 \dots]$.

Problem 1.12 Strongly Degenerate Expression for S

Show that the strongly degenerate expression for the entropy is given by $S = (N\epsilon_F/T)[(\pi^2/2)(T/\epsilon_F)^2 - (\pi^4/20)(T/\epsilon_F)^4 \dots]$.

Chapter 2

A Simple Nuclear Equation of State

In the free nucleon gas the particles interact with one another only through the exclusion principle embedded in the Fermi-Dirac statistics. In real nuclei, however, the strong and Coulomb forces also play an important role in the stability of the nucleus. To build a working model of the nucleus, the effects of the interactions must be added to the free nucleon gas. As there is not yet a first-principles way of solving the many body problem with strong interactions, it is necessary to resort to a phenomenological approach.

Most models simply add an interaction energy to the free nucleon gas. In order to represent a nucleus, the energy per nucleon at $T = 0$ should have a value of $\varepsilon(n_0) = -8 \text{ MeV}$ at saturation density n_0 . Also, since a nucleus is stable at this density, the pressure should vanish, and as observed in experiments of nuclear oscillations [Youngblood *et al.* (1984)], the compressibility should be of the order $100 \text{ MeV} < K(n_0) < 300 \text{ MeV}$. Several parametrizations of the interaction energy are able to reproduce these values (*eg.* [Danielewicz (1979)], [Bertsch and Siemens (1984); López and Siemens (1984)], [Kapusta (1984)], and others [Goodman *et al.* (1984)]), for simplicity the equation of state proposed by Kapusta will be used with the doubly quadratic fit of section 1.5 to mimic the temperature dependence of the free nucleon gas component.

2.1 The Nuclear Equation of State

The equation of state (*EOS*) of a medium formally relates the pressure with two independent variables, commonly taken as the density and the temperature. This expression for $p(n, T)$, can be obtained from the energy

of the system through $p(n, T) = n^2(\partial\epsilon(n, T)/\partial n)_S$. Adding an interaction energy $\epsilon_{int}(n)$ to the free nucleon gas, the energy per nucleon (measured with respect to the $T = 0$ value) is thus given by

$$\epsilon(n, T) = \epsilon_{int}(n) + [\epsilon_F(n, T) - \epsilon_F(n, 0)] = \epsilon_0(n) + \epsilon_T(n, T) .$$

Notice that in this simple model, the interaction energy per particle is assumed to be temperature independent and denoted by $\epsilon_{int}(n) = \epsilon_0(n)$. The thermal part of the energy per nucleon, ϵ_T , is given in terms of the free nucleon gas energy, $\epsilon_F = E_F/N$, (cf. Eq. (1.6)) and its zero-temperature value, $\epsilon_F(n, 0) = 3\mu_0(n)/5$ (cf. problem 1.2).

The parametrization introduced by [Kapusta (1984)] sets the ground state energy as $\epsilon_0(n) = \sum_{i=2}^5 a_i(n/n_0)^{i/3}$, with the parameters $\{a_i\}$ adjusted to yield $p = 0$, $\epsilon_0 = -8 \text{ MeV}$, $K = 210 \text{ MeV}$ and the correct zero-temperature value of the Fermi gas energy, all at saturation density. [The isothermal compressibility is obtained from $K = n(\partial p/\partial n)_T$.] The corresponding equation of state is thus

$$p(n, T) = \frac{n_0}{3} \sum_{i=2}^5 i a_i \left(\frac{n}{n_0} \right)^{i/3+1} + \epsilon_T(n, T) , \quad (2.1)$$

with $a_2 = 21.1$, $a_3 = -38.3$, $a_4 = -26.7$, and $a_5 = 35.9$ all in units of MeV . To further simplify this expression, one can use the fit presented in the previous section, namely $\epsilon_T(n, T) = \sum_{i=0}^2 \epsilon_{Ti}(T)n^i$ with the temperature-dependent coefficients of ϵ_{Ti} listed in table 1.1. Figure 2.1 shows the isothermal pressure curves obtained with equation (2.1) as a function of the number density.

As seen in figure 2.1, the pressure isotherms have a behavior characteristic of classical media. Normally, a reduction of the volume occupied by a gas, i.e. an increase of the density, increases the pressure since $\partial p/\partial n > 0$. Figure 2.1, however, shows the existence of a region where $\partial p/\partial n < 0$. In this region the compressibility is negative, and the system responds to an increase of the density with a reduction of the pressure and a further increase of the density. This is not a stable region, as those density fluctuations that normally occur in any continuous media will break the system into pieces of low density (gaseous particles) and high density (liquid particles). These regions are known as the isothermal (if $(\partial p/\partial n)_T < 0$) and adiabatic (for $(\partial p/\partial n)_S < 0$) *spinodal* regions.

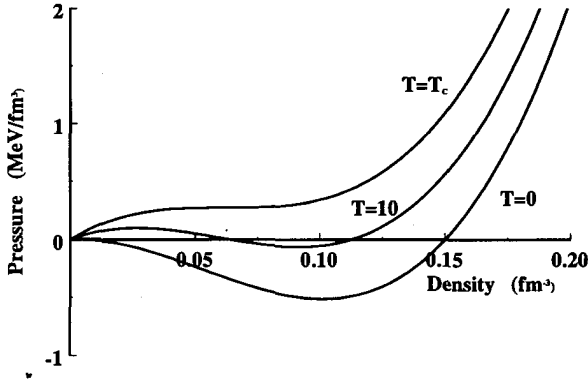


Fig. 2.1 Isothermal pressure curves as a function of the density for three temperatures, with $T_c = 14.542 \text{ MeV}$.

But this is not the whole story, even before a nucleus enters the spinodals, there is a region where, although $\partial p / \partial n > 0$, no extra energy is needed remove a nucleon from a nucleus, or to bind one more to a free nucleon. Inside this region, known as the *coexistence* region, uniform nuclear matter abandons the pressure isotherms and breaks into phases adopting a pressure determined by the temperature and the average density of the system. This section will study the coexistence, isothermal and spinodal regions.

Problem 2.1 The Coefficient a_2

In the strongly degenerate case, the zero-temperature part of the Fermi gas energy goes like $E_F(n, T = 0) \sim n^{2/3}$. Calculate $E_F(n, 0)$ and compare the coefficient of $n^{2/3}$ to the EOS parameter a_2 .

2.2 Coexistence Curve

Inside the coexistence region the liquid and gaseous phases can be in equilibrium with one another. When this happens $T_{\text{liquid}} = T_{\text{gas}}$, $p_{\text{liquid}} = p_{\text{gas}}$ and $\mu_{\text{liquid}} = \mu_{\text{gas}}$. Unfortunately these conditions show no special geomet-

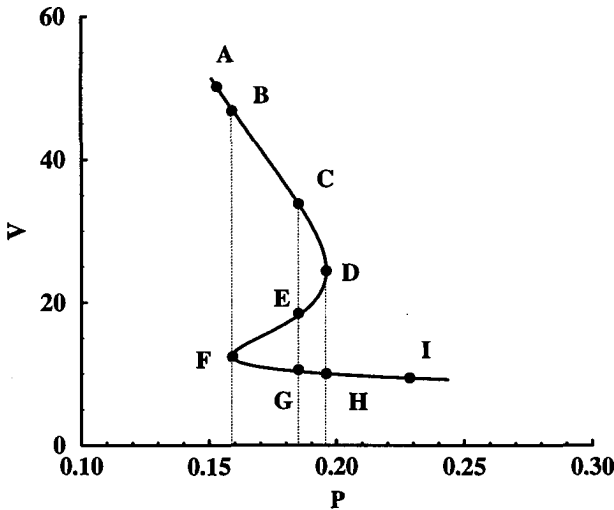


Fig. 2.2 Pressure Isotherm for $T = 13 \text{ MeV}$ plotted sideways versus the volume and showing the unstable region between points D and F .

rical features in the pressure curves and the boundary of the coexistence region must be determined by a special technique known as the *Maxwell construction*. Figure 2.2 shows the pressure isotherm for $T = 13 \text{ MeV}$ plotted sideways against the volume. The unstable region goes from point D to point F . To determine the boundary of the coexistence region it is necessary to look at the energy needed to add or subtract a nucleon from either phase, *i.e.* the Gibbs free energy, $g = \mu = \epsilon - Ts + pV$.

Infinitesimal changes of the Gibbs free energy per particle are given by $dg = -sdT + Vdp$. In an isotherm, where $dT = 0$, the change in the Gibbs free energy per particle is the area below the curve $V(p)dp$. To determine the boundary of the coexistence region, this area must be equal to zero.

Taking figure 2.2 as a reference, the area of interest is the one below the curve. Due to the change in the sign of dp along the loop, $(g_2 - g_1)$ is not a monotonous function of the limits. Then the corresponding expression for our *EOS*, in terms of the temperature and the volume, reads:

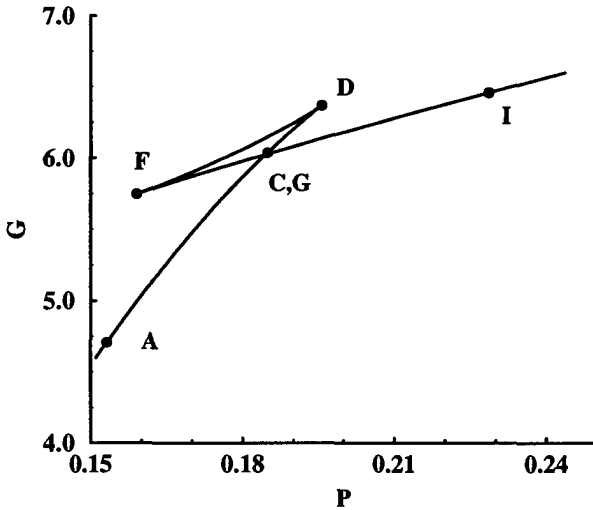


Fig. 2.3 g as a function of p for our EOS at $T = 13 \text{ MeV}$, i.e. just below T_c

$$\begin{aligned}
 g_2 - g_1 &= \int_{p_1}^{p_2} V(p) dp = \\
 &= \left[\left(\frac{5a_2}{3n_0^{2/3}} + \frac{8a_5}{3n_0^{5/3}V} \right) \frac{1}{V^{2/3}} + \right. \\
 &\quad \left. - \frac{2}{3} (\varepsilon_{01}^T + \varepsilon_{02}^T T) T \log(V) + \frac{7a_4}{3n_0^{4/3}} \frac{1}{V^{4/3}} \right. \\
 &\quad \left. + \frac{6a_3/n_0 + 4\varepsilon_{11}^T T + 4\varepsilon_{12}^T T^2}{3V} + \frac{\varepsilon_{21}^T T + \varepsilon_{22}^T T^2}{V^2} \right]_1^2
 \end{aligned}$$

where the ε_{ij}^T are given in table 1.1.

Figure 2.3 displays the behavior of g as a function of P for our EOS at a temperature of 13 MeV , i.e. below T_c . For a system to be in equilibrium g should be a minimum. So the allowed values of g are the ones lying on the curve $ACGI$. The points C and G are coincident, and can be determined

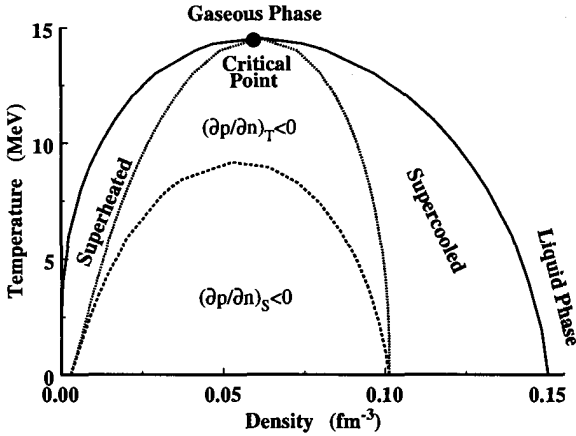


Fig. 2.4 Phase diagram of nuclear matter

by the condition $0 = \int_{p_c}^{p_g} V(p) dp$, which implies that

$$\int_{p_c}^{p_d} V(p) dp - \int_{p_e}^{p_d} V(p) dp = \int_{p_f}^{p_e} V(p) dp - \int_{p_f}^{p_g} V(p) dp .$$

This states that the areas determined by the curve of constant pressure p and the isotherm under study, to the left and the right of the intersect in the unstable region, must be equal. In this way the set of solutions of this “equal areas” condition determine the so called coexistence curve.

It is clear that the endpoints of the Maxwell construction can also be calculated from the conditions $(p_c = p_g)_{T,p}$ and $(\mu_c = \mu_g)_{T,p}$ since, in equilibrium, the pressures and the chemical potential of both phases are to be equal. The portion of the isotherms from the boundary of the coexistence curve to the limits of the unstable spinodal region are physically accessible, and are called the *superheated vapor* and *supercooled liquid* (cf. figure 2.4).

The **Mathematica** notebook **Coexistence.nb** listed in appendix B.3, is a pedestrian but illustrative (and fun) way to determine the boundary of the coexistence region.

2.3 Spinodal Regions

The dotted line of figure 2.1 shows the boundary of the unstable region, this is known as the isothermal spinodal line. This curve was calculated solving $(\partial p(n, T)/\partial n)_T = 0$ for n for a number of temperatures. The right branch of the curve shows the points where a further increase of the liquid density results in no change of pressure. The left branch of the curve shows the corresponding points for the gaseous phase.

The left and right branches of the isothermal spinodal meet at the critical point (n_c, T_c) . For the particular parametrization of equation (2.1), the critical point occurs at $n_c = 0.061 \text{ fm}^{-3}$ and $T_c = 14.542 \text{ MeV}$.

For adiabatic processes, *i.e.* those conserving the total entropy of the system, the nuclear medium can be described by the entropy counterpart of equation (2.1), $p(n, S) = n_0/3 \sum_{i=2}^5 ia_i(n/n_0)^{i/3+1} + \varepsilon_S(n, S)$, with $\varepsilon_S(n, S)$ given as in section 1.5. Again, the isentropic pressure $p(n, S)$ has a zone of negative compressibility (now the isentropic compressibility) known as the mechanically unstable region. This boundary, known as the *adiabatic spinodal* line, is outlined in figure 2.4 with a dashed curve. For this particular parametrization the end-point of the spinodal is located at $n = 0.034 \text{ fm}^{-3}$ corresponding to an entropy of $S = 2.145$.

Problem 2.2 Boundary of the Unstable Region

Use the equation of state (2.1) to determine the critical point and several points of the isothermal spinodal up to four significant digits. Hint: the condition $(\partial p(n, T)/\partial n)_T = 0$ yields the requirement:

$$0 = 5a_2n_0 (n/n_0)^{2/3} + 9a_3n + 14a_4n (n/n_0)^{1/3} + 20a_5n (n/n_0)^{2/3} + 3\varepsilon_{T0}n_0 + 6\varepsilon_{T1}n_0n + 9\varepsilon_{T2}n_0n^2.$$

Problem 2.3 The Mechanically Unstable Region

Repeat problem 2.2 to determine the end-point and several points of the adiabatic spinodal. Hint: the condition $(\partial p(n, S)/\partial n)_S = 0$ yields the same requirement as in problem 2.2 with the set $\{\varepsilon_S\}$ replacing $\{\varepsilon_T\}$.

Problem 2.4 The Entropy at T_c

Using the T -dependent equation of state (2.1) and its entropy counterpart, determine the entropy of the critical point. Hint: solve $p(n_c, T_c) = p(n_c, S_c)$ for S_c . What is the temperature of the end-point of the spinodal?

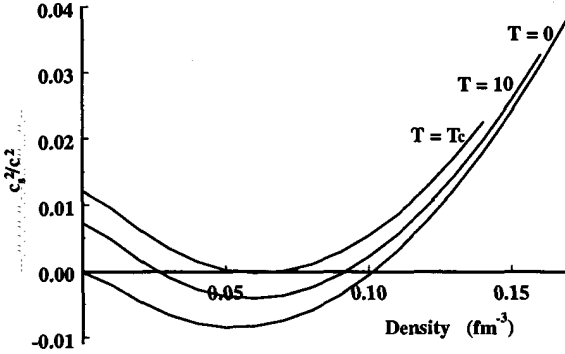


Fig. 2.5 Isothermal speed of sound squared normalized by c^2 as a function of n for $T = 0, 10$, and $T_c = 14.542 \text{ MeV}$.

2.4 Speed of Sound

The behavior of sound is deeply connected to the spinodal regions. Due to the ill-behaved speed of sound inside the unstable zone, it is better to study the square of the speed of sound, $c_s^2 = (\partial p / \partial n)_T / m$, which for our equation of state is

$$c_s^2 = \frac{2}{9mn_0} \left[9a_3n + 14a_4n \left(\frac{n}{n_0} \right)^{1/3} + 5a_2n_0 \left(\frac{n}{n_0} \right)^{2/3} + 20a_5n \left(\frac{n}{n_0} \right)^{2/3} + 3n_0\epsilon_{T0} + 6n_0n\epsilon_{T1} + 9n_0n^2\epsilon_{T2} \right] \quad (2.2)$$

where the ϵ_{Ti} are listed in table 1.1. A couple of observations are in order, first, if m is given in MeV/c^2 , c_s^2 will be in units of c^2 , the speed of light squared. Second, in deriving expression (2.2), T has been held constant to yield the isothermal speed of sound, the adiabatic speed is given by a similar expression with T replaced by S , and the functions ϵ_{Ti} by ϵ_{Si} .

The peculiar behavior of c_s^2 is shown in figure 2.5. For a number of densities and temperatures $c_s^2 < 0$, the region where this happens corresponds to the isothermal spinodal, as discussed in section 2.3.

2.5 More Conclusions

Here the free nucleon gas was dressed up with nucleon-nucleon interactions to build a more realistic model of nuclear matter. The Kapusta parametrization of the interaction energy was coupled with the doubly quadratic fit of section 1.5 to produce an interacting nucleon gas model. Cold nuclear matter at saturation density ended with a binding energy per particle of $\varepsilon(n_0) = -8 \text{ MeV}$, zero pressure, and a compressibility of $K(n_0) < 210 \text{ MeV}$.

The resulting model allows an easy calculation of isothermal and isentropic pressure curves as a function of the number density. These calculations show the existence of liquid and gaseous phases, as well as a *coexistence* region and unstable isothermal and adiabatic *spinodal* regions. For the equation of state used, the critical point occurred at $n_c = 0.061 \text{ fm}^{-3}$ and $T_c = 14.542 \text{ MeV}$, and the end-point of the adiabatic spinodal at $n = 0.034 \text{ fm}^{-3}$ and $S = 2.145$. The model also allowed an easy parametric calculation of the speed of sound which again showed the unstable regions.

This model will now be used in a more in-depth study of the dynamics of phase changes.

2.6 Additional Problems

Problem 2.5 Saturation Density

Using expression (2.1) verify that nuclear matter is stable at saturation density n_0 , i.e. verify that $p = 0$ in the ground state at n_0 .

Problem 2.6 Nuclear Compressibility

Derive an expression for the isothermal compressibility K starting from expression (2.1), and verify that $K = 210 \text{ MeV}$ at the ground state at saturation density.

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Chapter 3

The Road Toward Mixed Phases

Nuclear matter can go from a uniform phase to a mixed phase if it enters the coexistence region (*cf.* figure 2.4). The passage from a continuous phase to a liquid-gas mixture can happen in several ways depending on the initial density and temperature (or entropy) of the nucleus, as well as on the speed of the expansion. Here the isentropic expansion of a nucleus is reviewed first. This exercise will identify the possible routes to a phase change namely, evaporation-condensation, liquid-to-gas phase transition, and isothermal and adiabatic spinodal decomposition. These possibilities will be examined in the rest of the chapter.

3.1 Isentropic Expansion

So far the existence of phases has been discussed, but not the processes leading to the formation of these. This section presents a brief description of the thermodynamics of expanding nuclear matter.

As seen in figures 2.1 and 2.4, both the isentropic and isothermal spinodals are inside the region where bubbles and droplets can coexist. A nuclear system expanding past these boundaries will respond with *negative compressibility* enlarging any small density fluctuation and disassembling the system into droplets.

An idealized collision between two large heavy nuclear ions could, in principle, form a compressed and heated system that might expand after the initial collision. [In terms of density, temperature and entropy, the medium would go from $(n, T, S) = (n_0, 0, 0)$ to some point with $n > n_0$, $T > 0$, and $S > 0$.] Since this hot and dense system is now isolated from the

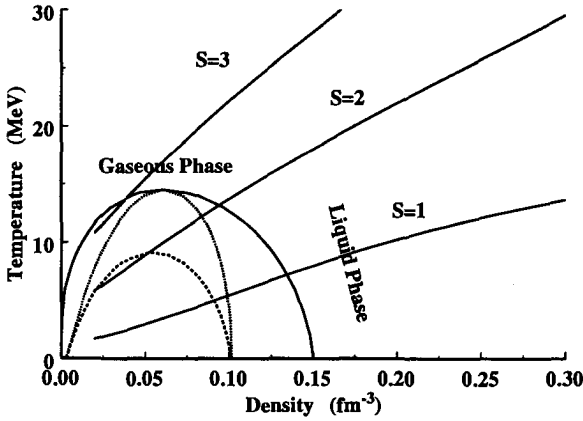


Fig. 3.1 Isentropic Expansion of Nuclear Matter

rest of the universe, it would expand adiabatically into vacuum conserving the total entropy generated during the impact. As the density of the blob drops, its temperature will decrease, but as long as the system remains in a single phase there will be no further changes of entropy. Its temperature evolution can thus be mapped to the n - T plane on a constant entropy path.

Since there is a one-to-one correspondence between any pair of points (n, T) and (n, S) , one can solve for T from $p(n, T) = p(n, S)$, which, in our parametrization, is equivalent to $\varepsilon_T(n, T) = \varepsilon_S(n, S)$. The resulting expression is

$$T(n, S) = -T_B(n) + \frac{\sqrt{T_B^2(n) + 4 T_A(n) \varepsilon_S(n, S)}}{2 T_A(n)}, \quad (3.1)$$

where, as in section 1.3, $T_A(n) = \sum_{i=0}^2 \varepsilon_{i2}^T n^i$ and $T_B(n) = \sum_{i=0}^2 \varepsilon_{i1}^T n^i$. Figure 3.1 shows the trajectories that expanding systems at $S = 1, 2$, and 3 would follow on the $n - T$ plane. Once the expanding nuclear system enters beyond the boundary of the coexistence region, it will decompose into liquid and gaseous phases. The final configuration will depend on the speed of the expansion as well as on the isentrope followed during the expansion.