

ON THE PULSAR

B. B. Kadomsev

Foreword by **A. Nomerotsky**

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formerly of Russian Academy of Sciences

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Foreword

This book about pulsars, by Academician Kadomtsev, may be surprising to people in the future because of the insights it provides into the field. He has combined an excellent introduction to pulsar physics with reflections on important informational and biological issues, all the while fusing this into a personal approach to educating the reader.

At first, the book appears to be a ‘pop’ book on physics but only for a few moments. The book contains two main strands: the first concerning the physics and the second dealing with a narrative about two boys that an old professor takes for a journey into the unknown. The story evolves in using the dialogues of Professor Leonid Andreevich with two teenagers who have a budding interest in science, Sasha and Misha. At times the story sounds like a piece of fiction, it has recollections from childhood, philosophical deviations and personal comments. Some of these tales are an interesting reflection on the period in which Kadomtsev was writing the book at the end of last century, when Russia was going through an historic transition from socialism to capitalism.

Pulsars are the product of neutron stars, so this topic explains in close detail the behaviour of matter at extremely high density, when all nucleons are close together, thus creating a strong magnetic field. Professor Kadomtsev likes to start his explanations with simple things understandable by everybody, such as bouncing balls.

— Leonid Kadomtsev —

He then continues explaining the strange world of super strong magnetic fields, where mass is anisotropic and hydrogen atoms could form polymer chains.

Parts of the book are science fiction but they were deliberately designed so as to better illustrate the point in reference. In the beginning the reader will be able to tell the difference between that which is real and that which is fantasy but in the last two chapters the lines will not be so evident. There, the book takes an unusual turn and takes the reader to the wonders of self-organization and living organisms, topics which seem to be completely unrelated to the pulsar. The meaning of it becomes clear in the last section. Could the pulsar be sentient? To find out, the professor takes the boys to the pulsar itself and they find some evidence of self-organization, only to wake up to find it was a dream!

The level of the book is ambiguous. The author is driven by common sense and tries his best to make it an easy read. Kadomtsev believes that much of the book should be comprehensible to high school students. But, as a book written in Russia where the physics programme in schools and universities was, at the time, more advanced, the book therefore deals with topics which will not be familiar to a high school student in the US or Europe. The author tries to stretch the reader, to involve them in the process. His examples are highly original and revealing. The switches between simple and complicated topics occur very naturally during the dialogues; “what is the colour of an electron?” one of the students ask, “it is white”, the professor replies. This would never occur to me!

Appendices explain the more complex subjects and have an excellent collection of problems with solutions.

This book has an unforgettable spirit which may seem old-fashioned to some or too ‘soviet’ to others. We can see Kadomtsev building a temple of science in front of us, while he explores different halls and chambers, some of them well illuminated and other slightly murkier. The voices of people like Landau and Sakharov still resound in this temple. The book clearly reflects the scientific and

educational establishment of the Soviet Union during its last decades. This is what it was, and it is a system that had shortcomings, but at the same time produced scores of the brightest minds.

So go on, reader, explore and enjoy this book on the pulsar.

Andrei Nomerotski
Jesus College
University of Oxford

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Unexpected Encounter

Misha left the main building of Moscow State University. Today the University was open to the senior grade high school pupils, and they were welcome to visit the lecture rooms and laboratories. Misha was dazzled and confused by the endless number of instruments, machines and facilities.

Outside, Spring was in full swing. It was one of those days in May when the land is flooded with bright sunshine and the young greens sparkle with emerald light.

Misha sat on a bench trying to clear his head. A young girl was passing by. She was beautiful — in a glance he took in her large dark eyes, long lashes, and lovely face. He followed her with his eyes thinking of making her acquaintance. It would be great, for example, if some hooligan tried to attack her and he, Misha, would come to her rescue. And the girl would give him a smile . . .

Suddenly Misha felt someone looking at him. Only now did he notice that he was sitting next to a gentleman in his fifties. Slim and tidy, he was staring at Misha with smiling eyes.

Misha flushed. He didn't like old people: they were usually very demanding and liked to interfere in everyone else's business.

"Young man, please do not look at an old and distinguished academic as if he's a dog," uttered the gentleman kindly, but firmly, rapping out the words. "When I was your age I used to think that all people over forty were stupid old fellows who were out of their minds

and needed to be retired. As you grow older you start viewing age differently. In England, for example, people over 60 are considered to be middle-aged. Well, I have not introduced myself yet,” added the stranger. “I am Professor Leonid Andreevich Petrov. How should I address you?”

“Misha,” mumbled Misha, very confused by meeting a famous scientist and by his familiar yet respectful manner.

“Misha is a very good name,” said Leonid Andreevich. “Well, Misha, I heard you asking questions during the tour of the University. I liked your questions. Curiosity is very important in science, and so is the ability to ask questions, even if they are still not very sophisticated ones. Very often a fresh, inexperienced glance grasps more than the persistent and patient study of a professor who can’t see beyond the end of his own nose.

“Well, Misha, I am busy with an investigation where a keen, inquisitive and direct mind is required, and you seem to possess all these qualities. Therefore, I invite you to help me. The summer vacation has already started, so you may have some spare time. If you’re interested, please come tomorrow at ten o’clock in the morning to this address.” with that the Professor gave Misha a business card with his office address. “You’ll need to bring your ID card so that you can enter the building.”

“All right,” said Misha, and without adding anything else he shook the hand of the smiling Professor.

Next morning, at ten minutes to ten, Misha was standing at the entrance to a small building with an engraved plate: ‘Institute of Systems Studies of the Academy of Sciences.’ Misha gingerly entered the lobby and joined the queue of people waiting to be allowed in by the guard. Everyone showed him their passport and the guard, after a cursory glance at the name, took his time scrutinising a small piece of paper with something written on it. Now it was his turn: after a swift glance at his ID card, the guard carefully studied his pass as if trying to find some mistake, then handed it to Misha and said: ‘Room 201, second floor.’

The door of room 201 bore a nameplate: “Professor Leonid Andreevich Petrov.” Misha knocked at the door and opened it, but instead of the Professor he saw a young woman at a desk. The woman was pretty, but had an austere and discontented look about her. Misha was immediately reminded of his English teacher.

“Good morning, I was invited by Leonid Andreevich,” said Misha and gave his name. Suddenly the face of the young woman — and Misha understood that she was the Professor’s secretary — changed completely, expressing a warm welcome.

“Leonid Andreevich is expecting you.” She pointed at a huge oak door.

Misha opened the door and saw Leonid Andreevich smiling at him with a telephone in his hand.

“Here you are, professor,” joked Leonid Andreevich interrupting his conversation. “Take a seat while I finish my call.”

While Leonid Andreevich was talking, Misha examined his room. It was a rectangular room with two tables — a desk which was occupied by Leonid Andreevich and a large table with chairs for conferences. Opposite the conference table there was a blackboard and some posters. One of the posters, beautifully made, read: ‘Study of a pulsar’ and below there was some kind of a diagram.

“Hello, Misha,” Leonid Andreevich said merrily. He took a seat opposite Misha and said: “Yes, young man” — as if inviting him to start a conversation, and, quite unexpectedly, Misha asked “What is a pulsar?”

“That is a good question. Let us start with it.”

And he told Misha the following.

What is a Pulsar?

In 1967 a powerful radiotelescope was built in Cambridge to study the interplanetary scintillation of radiowaves. Radiowave scintillations themselves were discovered somewhat earlier in 1964. Let us suppose that there is a radiowave source located very far from the Earth. The radiowaves travel long distances before they reach the Earth's surface, and on the way they pass through clouds of rarefied plasma in interplanetary space. These diffract the waves, and so scintillations occur on these clouds. The study of interplanetary scintillations allows us to obtain very important data on the angular dimensions of a radiosource.

Usually, when speaking of a radiotelescope, we imagine a huge parabolic mirror which, like a gigantic eye, is directed into the depths of space. However, the Cambridge telescope, or, to be more precise, the telescope of the Mullard Radio Astronomy Observatory, looked completely different. This huge structure resembled the eye of a fly or a dragon-fly, and consisted of more than two thousand dipole antennae. They filled a rectangular area of about eighteen thousand square meters. All the dipoles were precisely phased, so that they formed a 'compound eye'. By changing the phases of the dipoles, the 'direction' of the antennae could be changed also, i.e. the 'compound eye' could be rotated.

Routine operation began in July, 1967. The telescope permitted simultaneous observation of several different sectors of the sky. At the end of August, Miss Bell, whilst checking the recorded data, found a very peculiar dot. It looked like a scintillating source which flickered at about midnight when interplanetary scintillations were weak. As the amount of data grew, it became evident that the source was located far beyond the Solar System. The possibility of radiodisturbances from a far space object was excluded.

Further observations showed that this radiation was emitted in periodic pulses. You can imagine the excitement of Professor A. Hewish, Miss Bell and their colleagues, who encountered this new phenomenon. Estimates of each pulse duration showed that they were no longer than twenty milliseconds. This proved that the source was of planetary dimensions; the possibility that the signals came from an alien civilization couldn't be ruled out. The observations revealed that the pulses had an amazing regularity. Their repetition frequency was precise to better than 1 part in 10^7 .

The search for similar objects very soon resulted in the discovery of three other pulsing objects. They were named pulsars. This was reported in 1968 and, starting from February, 1968, all the major radiotelescopes began to study these new space objects. For his role in the discovery of pulsars, Professor Hewish was awarded the Nobel Prize. Later many other pulsars were discovered. They all have an exceptionally precise repetition of pulses, with periods that range from several milliseconds to several seconds.

The natural explanation of a pulsar's precise repetition is that it is related to a star's rotation. In this case the star should be very compact. The existence of such stars had already been theoretically predicted, and they were called neutron stars. The substance of neutron stars resembles that of the nuclei of heavy atoms.

According to theory, neutron stars can be formed at the end of the evolution of massive stars. After the nuclear fuel of the star 'burns out', i.e. the majority of nuclei are Fe nuclei, the central part of the star gets compressed. The density grows and at a certain moment

Fe nuclei, which are located close to each other, disintegrate, which is very energetically favorable. The star's core collapses inwards at high velocity. At the same time, the star's shell is ejected: a supernova has occurred.

After such a flash, the core of the massive star implodes to a 'tiny' neutron star (its radius is only several kilometers), around which a swiftly expanding shell forms. As it moves away from the star, this shell transforms into a nebula. The most well-known nebula of this type is the Crab Nebula, and just inside this nebula one of the pulsars was discovered, or, to be more precise, a short-period pulsar, i.e. a relatively young neutron star.

Why does such a star emit radiowaves? It turns out that a neutron star is magnetized. Like the Earth, it has a dipole magnetic field. The only difference is that it is a hundred billion times stronger than the magnetic field of the Earth. As it happens, the superstrong magnetic field of a neutron star may drastically change the properties of matter. Additionally, this gigantic magnet, rotating very quickly, significantly excites the surrounding plasma. This very excited plasma becomes a radiotransmitter which generates radiowaves. The period of repetition of radiopulses is exactly equal to the period of the neutron star.

Simply speaking, a pulsar is a unique object in Nature!

.....

Mischievous Ball

“And now we shall go and see what a pulsar is,” said Leonid Andreevich.

“What do you mean?” inquired Misha.

“You will find out very soon,” answered Leonid Andreevich, and he invited Misha to follow him.

They left the Professor’s room and went to the very end of the corridor. Leonid Andreevich stopped near a large door and pressed some buttons on the coded lock. The door slowly moved aside and they stepped in. Misha never expected to find such a room in the building: they came into a huge round hall with a hemispherical dome which was evenly illuminated by daylight. To tell the truth, Misha thought that they were outside the building, and only after several moments did he understand that they were still indoors. The hall reminded him of the Planetarium, the only difference being that there were no chairs on the periphery. In the centre of the hall there was a large round table instead of the projector that showed the sky and stars. Around the table there were several chairs which looked like dentist’s chairs: they could be turned and reclined so that it was very easy to look upwards. Opposite one of the chairs, a large display was installed and it had a computer keyboard beside it. Numerous different things were scattered all over the table. Among them Misha noticed some large glass plates and a box with balls which looked

like ping-pong balls. Leonid Andreevich took the chair opposite the display and invited Misha to take a seat next to him. The display showed one line of text, ‘Earth, day, cloudy’ and some numbers.

Leonid Andreevich pressed one of the keys. The display went blank and he typed ‘Pulsar’ and then some numbers and pressed the same key. Suddenly the hall was transformed. The floor around the table was emitting a bright blue light and the dome had become a black night sky with myriad shining stars.

“We are on a pulsar already”, said Leonid Andreevich. “Now we shall do some experiments, and, to make it easier to see, I shall switch on the Sun, which, of course, you would not be able to find on a pulsar. For our comfort I had better hide it behind some clouds.”

Leonid Andreevich keyed in some numbers and a bright daylight filled the hall again.

“We shall start with the simplest experiments,” said Leonid Andreevich. He took one of the balls from the box, lifted it above the glass plate on the table and let it drop. The ball started to bounce, the height of each bounce slowly decreasing until, after some time, the ball came to rest on the plate.

“This is a very good ball, it bounces for quite a long time,” explained Leonid Andreevich.

“If the ball did not lose energy at each bounce, it would continue bouncing forever. Do you think that if we observed these bounces of smaller and smaller amplitude through a microscope, we’d see that this ball bounced forever too?” asked Leonid Andreevich.

“I don’t know,” answered Misha, puzzled.

“Well, let us think. Let us assume that at any stroke the ball loses a small fraction of its momentum. Let us denote this fraction by ϵ . In other words, if the ball’s velocity before a stroke is v , then after a stroke it will be equal to $v - \epsilon v$. We assume that there is no air resistance. Then, before the second stroke the velocity will be the same, i.e. $(1 - \epsilon)v$, and after the second stroke $(1 - \epsilon)(1 - \epsilon)v$. So, after the n th-stroke, it is equal to $(1 - \epsilon)^n v$. If the ball is initially thrown upwards from the surface with velocity v_0 , then it will drop

back after $t_0 = 2v_0/g$, where g is the free-fall acceleration. The time of the subsequent bounce will be $t_1 = 2v_1/g = 2(1 - \varepsilon)v_0/g$, and, consequently, $t_n = 2(1 - \varepsilon)^n v_0/g$. Thus, the total time of the ball's bounces is equal to

$$t = \sum 2(1 - \varepsilon)^n v_0/g$$

where the $\sum$ symbol denotes the sum over all n , beginning from $n = 0$ and ending at infinity.

“How can we estimate this sum? It is very simple. Let us say that $x = (1 - \varepsilon)$. We wish to find the value of $A = \sum_{n=0}^{\infty} x^n$. Multiplying A by $1 - x$, let us write down the result in two lines, one above the other.

$$\begin{aligned} (1 - x)A &= 1 + x + x^2 + \cdots \\ &\quad -x - x^2 - \cdots \end{aligned}$$

We see that the second line added to the first line gives unity. It follows that $(1 - x)A = 1$, i.e. $A = 1/(1 - x)$. Thus, the total time is

$$t = \frac{2v_0}{g} \frac{1}{1 - x} = \frac{2v_0}{g\varepsilon}$$

We have certainly treated infinite series rather freely, but actually the trick of summation of infinite series that we have used accords with the results of more sophisticated mathematics.

“Thus, we have found out that the ball will bounce only within a limited time t , after which it stops completely.

“By the way, this postulate solves one of Zeno’s famous ‘aporia’. In his time the Greek philosopher Zeno puzzled his compatriots by unusual statements which he called ‘aporia’. One of them was, ‘Achilles can never catch up with a tortoise’. Here is how he was thinking:

“Let Achilles start running after the tortoise at some given time. When he reaches the place from which the tortoise started, it will have moved ahead a little bit. Achilles must reach this new point to catch the tortoise, but by the time he reaches it the tortoise will have moved farther away, and so on. It seems that Achilles will never catch the tortoise.

“The solution is that although it takes an infinite number of these steps for Achilles to catch the tortoise, the total time is definitely finite, so Achilles will catch up with the tortoise rather quickly. However, as Frenchmen say, *revenons à nos moutons* . . .

“Up until now we have been throwing the ball on Earth. Now let us go to the pulsar,” and the Professor once more played with the computer keyboard, then he took the ball, lifted it over the glass plate, and released it. To Misha’s amazement the ball started to fall at an angle to the table, then hit the plate, and bounced back along the same inclined line and continued bouncing as if it were tied to an invisible thread. Finally, it came to rest. The Professor repeated the trick and said: “And now I am going to explain what just happened.

“The magnetic field of a pulsar is very strong, so it even affects the movements of regular, non-magnetic objects. The superhigh magnetic field produces a mass anisotropy: the mass along the magnetic field direction $m_{\parallel}$ maintains its value, i.e. $m_{\parallel} = m$, whereas across the magnetic field the mass $m_{\perp}$ considerably exceeds m . In other words, Newton’s law along the magnetic field keeps its original expression:

$$ma_{\parallel} = F_{\parallel}$$

where $a_{\parallel}$ is the acceleration along the magnetic field, and $F_{\parallel}$ is the force component along the magnetic field.

“Across the magnetic field Newton’s law is:

$$m_{\perp}a_{\perp} = F_{\perp}$$

It turns out that $m_{\perp} = m(1 + b)^2$ where b is equal to the magnetic field, B , divided by a certain constant, B_0 , of the order of 10^9 Gauss*. The magnetic field on the pulsar can reach and even exceed 10^{12} Gauss, so $m_{\perp}$ can be significantly larger than m , i.e. the mass anisotropy can be very high.

“Let us assume that on our pulsar $b^2 \gg 1$, since high anisotropy is typical of a pulsar, and yet is very unusual for us.

*1 Tesla = 10 000 Gauss. Throughout the text magnetic field strengths will be given in Gauss. However, the accompanying formulae require B to be expressed in terms of Tesla.

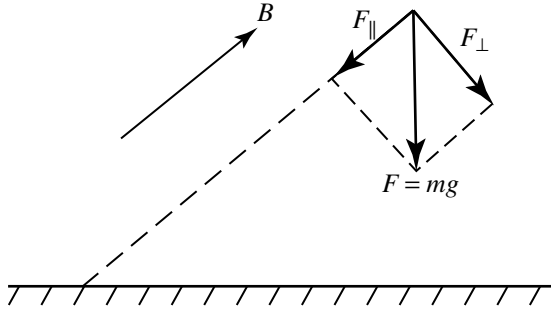


Fig. 3.1 The gravitational force $\mathbf{F} = m\mathbf{g}$ can be decomposed into two components: the longitudinal component $\mathbf{F}_{\parallel}$ is directed along the magnetic field $\mathbf{B}$, and the transverse one $\mathbf{F}_{\perp}$ is directed perpendicular to the field.

“Imagine that the magnetic field is inclined to the horizontal plane as shown in Fig. 3.1. The gravitational force, $F = mg$, is, of course, directed straight down. Note that this expression contains m , but not $m_{\perp}$. This is because the ball does not actually become heavier: the use of a $m_{\perp}$ value is simply a convention that accounts for the effects of the magnetic field.

“This force can be split into two components: along and across the magnetic field, i.e. $F_{\parallel}$ and $F_{\perp}$ respectively. Then Newton’s law can be applied:

$$m_{\parallel}a_{\parallel} = F_{\parallel}, \quad m_{\perp}a_{\perp} = F_{\perp}$$

As $m_{\perp} \gg m_{\parallel}$, the acceleration perpendicular to the magnetic field, $a_{\perp}$, turns out to be much less than the longitudinal acceleration. That is why the ball ‘slides’ almost along the magnetic field. We can easily find the distance, S , between where the ball lands and the intersection of the magnetic field with the horizontal plane. Indeed, if the initial velocity is zero, then the path length L along the field is given by the relation $L = F_{\parallel}t^2/2m_{\parallel}$, where t is the time of the fall. The transverse deviation is equal to $S = F_{\perp}t^2/2m_{\perp}$ so that:

$$\frac{S}{L} = \frac{F_{\perp}}{F_{\parallel}} \frac{m_{\parallel}}{m_{\perp}}$$

This relation is less than unity if the ratio of $F_{\perp}$ to $F_{\parallel}$ is not too high, i.e. if the angle between the magnetic field and the plane isn't too small.

“The values of the acceleration and velocity along and across the field differ greatly. Meanwhile, according to classical mechanics, the momenta $p_{\perp}$ and $p_{\parallel}$, are:

$$p_{\parallel} = m_{\parallel}v_{\parallel} = F_{\parallel}t, \quad p_{\perp} = m_{\perp}v_{\perp} = F_{\perp}t$$

It is easy to see that the total momentum lies in the same direction as the force due to gravity, i.e. vertically. Therefore, the momenta are not affected significantly by the mass anisotropy.

“Now let us try to understand what happens when the ball hits the horizontal plane. If the collision is elastic, the vertical component of momentum will change sign. Since there is no horizontal momentum component, this means that the velocities $v_{\parallel}$ and $v_{\perp}$ will change sign too: the ball will run backwards along the initial trajectory. The amplitudes of subsequent bounces will decrease due to small energy losses until the ball comes to rest.

“Now, let us consider another experiment. Let's throw the ball with a slight horizontal push. Now the collision point will drift along the plane between bounces since the horizontal component of the ball's momentum is nonzero. The resulting trajectory will look like that in Fig. 3.2a. It differs drastically from the analogous trajectory on Earth (Fig. 3.2b).

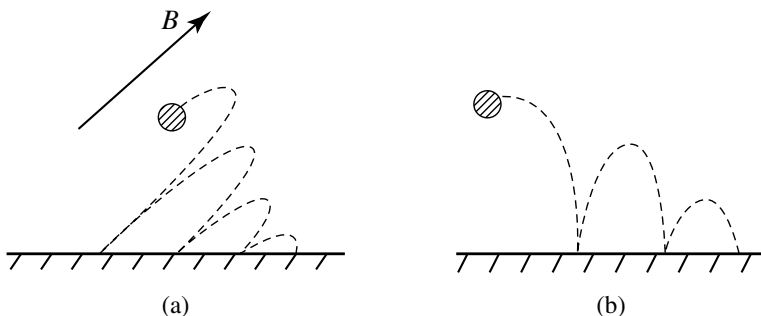


Fig. 3.2 A ball in a super-strong magnetic field bounces in a dramatically different way (a) to normal bouncing (b).

“First of all, note that a horizontal push to a ball on a pulsar causes not only horizontal, but also vertical displacement (this is impossible for an object with isotropic mass!). Besides that, it bounces radically differently from how it would on Earth. If it were on Earth, i.e. if the mass were isotropic, the angles on each side of a bounce would be equal because the horizontal momentum would be conserved (for elastic impacts), so the ball would bounce in the familiar manner shown in Fig. 3.2b. In the case of an anisotropic mass, the trajectory looks as is shown in Fig. 3.2a. From bounce to bounce the ball gradually shifts along the horizontal plane.”

“And what will happen if instead of a ball we take a bead on a stretched thread?” asked Misha.

“Oh, this is an excellent question,” answered the Professor. “I can even show you what will happen.”

The Professor rummaged in some boxes on the table, took out a string from one of them, pulled it through a big bead and then fastened the string at both ends so that it hung vertically. He lifted the bead and then dropped it. The bead started to slide along the string like in a slow motion film: it descended with constant acceleration, but at a much slower rate than in free fall.

“Let’s try to understand the reason behind this,” said L.A.

“Let the vector B of the magnetic field form an angle α with the string. A bead with a strongly anisotropic mass would tend to fall along the magnetic field line, but the string prevents it: a force N is produced by the string and is directed perpendicularly to the string. As a result, the bead will fall along the string with an acceleration a . It’s a simple problem in mechanics to find this acceleration if the forces are known. However, in our case we only know one of the forces — the force due to gravity. As for the force N , it is unknown, and we cannot ignore it.

“In solving similar problems, the dynamic task is often artificially reduced to a static one. For example, Newton’s second law for an isotropic mass can be written as:

$$\mathbf{F} + \mathbf{F}_i = 0$$

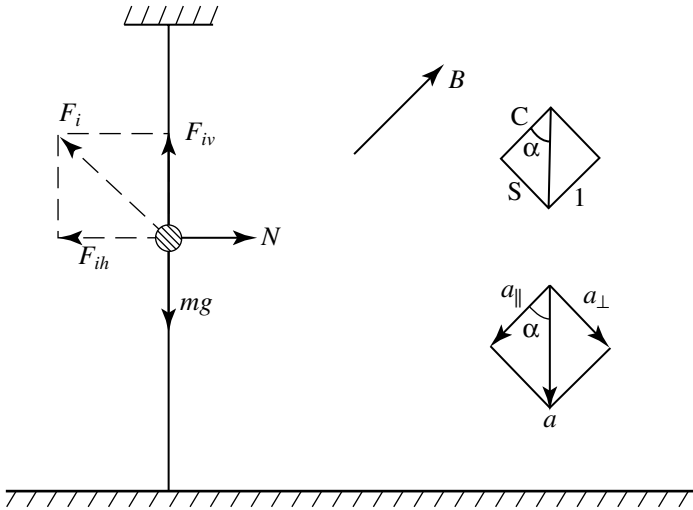


Fig. 3.3 When the motion of the bead is constrained by a string, the bead experiences a normal reaction force $\mathbf{N}$ in addition to the usual gravitational force $m\mathbf{g}$. In this case, the inertial force $\mathbf{F}_i$ has two components: a vertical one F_{iv} , and a horizontal one F_{ih} .

where $\mathbf{F}$ is the driving force and $\mathbf{F}_i$ is an artificially introduced inertial force which is equal to $\mathbf{F}_i = -m\mathbf{a}$. The net force, i.e. the sum of the forces $\mathbf{F}$ and $\mathbf{F}_i$, obeys the equilibrium condition. Let's use the same approach for our problem. If the acceleration of the bead is equal to a and is directed vertically down, it can be decomposed into two components: along and across the magnetic field (see Fig. 3.3).

Now we can find the components of the inertial force:

$$F_{i\parallel} = -m_{\parallel}a_{\parallel}, \quad F_{i\perp} = -m_{\perp}a_{\perp}$$

$\mathbf{F}_i$ is equal to the sum of the forces $\mathbf{F} = m\mathbf{g}$ and $\mathbf{N}$. Splitting $\mathbf{F}_i$ as shown in Fig. 3.3 into horizontal and vertical components, we find that the horizontal component balances $\mathbf{N}$, whilst the vertical component balances $m\mathbf{g}$. Therefore, if we are able to express F_{iv} in terms of the acceleration a , then, by setting this equal to mg , we will find the acceleration. If you know geometry this isn't a difficult problem, and it is even simpler if you know trigonometry.

"To further simplify our considerations, let's introduce an auxiliary rectangle with its sides along and across the magnetic field and

the diagonal equal to unity. The sides of this rectangle will be c and s , respectively (those who know trigonometry will immediately recognize that $c = \cos \alpha$, $s = \sin \alpha$, where α is the angle between the string and the magnetic field, so we will also use this more advanced notation).

“From the similarity of the rectangles — for $\mathbf{a}$ and for the auxiliary rectangle — we obtain:

$$a_{\parallel} = ac = a \cos \alpha, \quad a_{\perp} = as = a \sin \alpha$$

The inertial force $\mathbf{F}_i = -m_{\parallel}\mathbf{a}_{\parallel} - m_{\perp}\mathbf{a}_{\perp}$. The vertical component F_{iv} of this force is equal to the sum of the vertical components of each term. From the law of similar triangles we easily find:

$$F_{iv} = -m_{\parallel}a_{\parallel} \cos \alpha - m_{\perp}a_{\perp} \sin \alpha$$

Substituting in the values of $a_{\parallel}$ and $a_{\perp}$ we obtain: $F_{iv} = -(m_{\parallel} \cos^2 \alpha + m_{\perp} \sin^2 \alpha)a$. However, we derived this on the condition that $F_{iv} + mg = 0$. Therefore, we have:

$$(m_{\parallel} \cos^2 \alpha + m_{\perp} \sin^2 \alpha)a = mg$$

It looks as if the bead is getting heavier: its effective inertial mass is equal to $m_* = m_{\parallel} \cos^2 \alpha + m_{\perp} \sin^2 \alpha$. In the case of a very strong anisotropy, $m_{\perp} \gg m_{\parallel}$, the mass m_* appears to be much larger than $m_{\parallel}$, if the angle α isn't too small. The bead, therefore, falls along the string with an acceleration significantly lower than g : the fall appears to be moderated in time. Still, we have to make sure that the law of conservation of energy is satisfied. The velocity of the bead at time t is equal to $v = at$, where t is measured from the beginning of the fall with zero initial velocity. The distance S fallen through in time t is equal to $at^2/2$. So, if an object falls from height h , $S = h = at^2/2$. h may be expressed in terms of the velocity: $h = v^2/2a$. Since $a = gm/m_*$, we obtain: $\frac{1}{2}m_*v^2 = mgh$. So we see that the kinetic energy at the end of the path is equal to the potential energy at height h .

“We are now ready to describe the movement of a bead along a string inclined at an arbitrary angle to the horizontal. First, note

that our treatment of forces for a vertical string can also be used for the momentum components, i.e. if v is the particle velocity along the string, the longitudinal and transverse velocity components (along and across the magnetic field) are:

$$v_{\parallel} = v \cos \alpha, \quad v_{\perp} = v \sin \alpha$$

Let us consider the momentum of a particle $\mathbf{p} = m_{\parallel} \mathbf{v}_{\parallel} + m_{\perp} \mathbf{v}_{\perp}$. The momentum component along the string is equal to:

$$p = (m_{\parallel} \cos^2 \alpha + m_{\perp} \sin^2 \alpha) v = m_* v$$

As we know, acceleration is the rate of change of velocity. Thus, Newton's equation for motion along the string can be presented as a balance between the rate of change of momentum and the external force. The m_* value depends only on the orientation of the string with respect to the magnetic field, i.e. it can be used for the inclined string, minding the angle between the string and the magnetic field. The gravity force component along the inclined string will be less, of course, since it is well-known from the motion along the inclined plane. Further deliberations are simple enough for you to go on with them independently.

"Now we shall perform another experiment," said L.A.

"Let us take a ball and bounce it not on the horizontal plane, but on the plane perpendicular to the magnetic field. From Fig. 3.4 we can see that the ball, bouncing, slides along the plane. This case is simple enough, since the motions along and across the magnetic field are not connected with each other: the ball bounces along the magnetic field because the mass $m_{\parallel}$ is subjected to the component of the force of gravity along the field; it slides along the plane because the transverse mass $m_{\perp}$ is subjected to the transverse gravity component. As $m_{\parallel} \ll m_{\perp}$, the ball bounces rather frequently, but moves slowly across the plate."

"That wasn't hard to understand," said Misha, who followed L.A.'s deliberations only with some difficulty.

"Now let us do the following," said L.A.

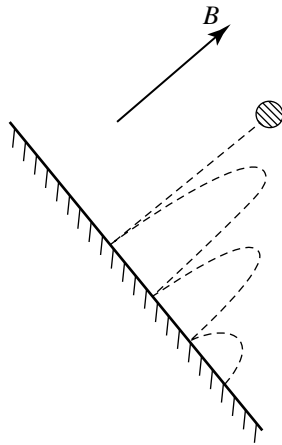


Fig. 3.4 Ball bouncing in the case when its mass is highly anisotropic due to the presence of a superstrong magnetic field B .

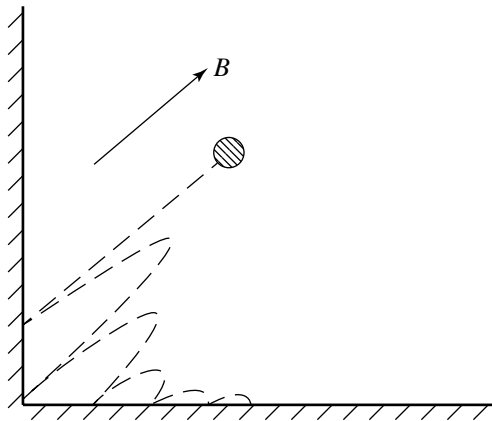


Fig. 3.5 Ball bouncing near a corner in the case of a very high mass anisotropy.

“Let us take a couple of plates and place them one vertically, the other horizontally at right angles to each other. Let the ball first fall on the vertical plate.” L.A. released the ball, and it fell along the force line in the direction of the vertical plate. Then, bouncing, the ball slipped down along this plate and rolled along the horizontal plane (Fig. 3.5).

“The ball gains horizontal momentum when it collides with the vertical plate. Part of the kinetic energy of the longitudinal motion is transferred into transverse (with respect to the magnetic field direction) kinetic energy. Thus, even in the ideal case of elastic collisions, the amplitude of the bounces along the magnetic field decreases gradually from stroke to stroke. At last, the ball descends along the wall so that its average fall looks like the bead motion along the vertical string. The ball possesses fairly large horizontal momentum when it reaches the horizontal plate. The ball starts bouncing as if it were thrown with an initial horizontal push.

“Well, you have been introduced to the simplest cases of motion with a highly anisotropic mass. This is the way that objects should move in a pulsar’s superstrong magnetic field.”

“Are we really on a pulsar?” Misha seemed to be surprised.

“Well, no.” L.A. smiled with the boy. “We are in a lab which can simulate the conditions on a pulsar. This hall is designed to study adaptation, i.e. the accommodation of a man to unusual external conditions. The walls and the dome of the hall are made of numerous LCD screens. These displays receive signals from a very powerful computer capable of reproducing a consistent picture from a great number of images.” “Here is an example.”

L.A. keyed in some command and Misha almost cried out with surprise: he found himself on the beach of a tropical island. The surf gently licked the sandy shore, the palm trees swayed in the breeze, and the air was filled with the sounds of the jungle.

“Now let us return to Russia.” said L.A., and in a couple of moments they found themselves on a lawn somewhere in the suburbs of Moscow.

“This round table is unusual too,” said L.A. “Inside it has a complicated system of coils through which currents are passed to produce alternating magnetic fields. The computer automatically controls these fields so that the balls, which are equipped with special internal structures, are capable of rather complicated motions. In particular, it is very easy to simulate the motion of objects with

anisotropic mass. We simulate only a portion of the conditions on the pulsar. This is why I speak of a ‘trip to the pulsar’. Later, we will introduce more variations into our experiments. Your task is simply to carefully observe everything, ask questions and try to understand what is going on.

“Well, that is enough for today,” added L.A. “Come back in a week’s time. Meanwhile, to keep you busy, I would like you to work through this at home,” — and he gave Misha several pages of printed material. The first page was titled: Problem No. 1 and it contained text with equations and figures (it is given in the Appendix).

Misha said goodbye and hurried home.

Misha and Sasha

Having spent some time at home, Misha decided to visit his friend Sasha. Sasha seemed very smart to him as he was a third year student at Moscow State University's Physics Department. Sasha was always ready to answer any question in physics and mathematics and Misha thought that he knew almost everything about physics. While speaking about his talk with the Professor, Misha briefly mentioned mass anisotropy in a strong magnetic field. Sasha's reaction was very abrupt:

“You must have mixed things up. There is no such thing as mass anisotropy. I have just studied the physics of magnetic phenomena and I know this field inside out!

“Almost all matter belongs to one of three classes of magnets — diamagnets, paramagnets, and ferromagnets. Diamagnets and paramagnets do not possess an intrinsic magnetic moment, they are only magnetized in the presence of a magnetic field. In diamagnets, an applied magnetic field gives rise to a magnetic moment which is directed against the field, whereas in paramagnets the induced magnetic moment is directed along the field. Paramagnets seem to consist of a multitude of elementary magnets which, in the absence of a magnetic field, are randomly oriented so that their total magnetic moment is equal to zero.

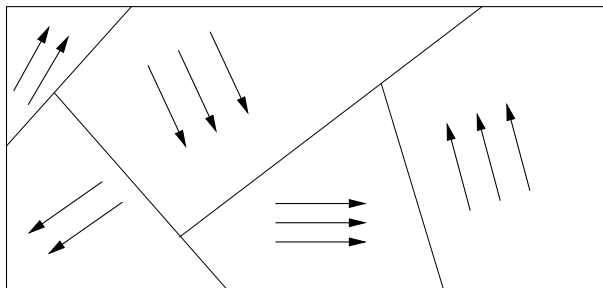


Fig. 4.1 Ferromagnets look like a composition of magnetic domains: each of them is magnetized up to saturation.

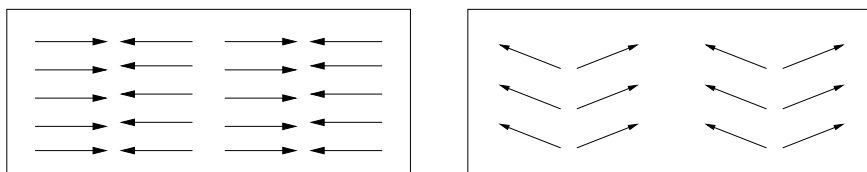


Fig. 4.2 Antiferromagnets (a) and weak ferromagnets (b) have two equal magnetic sub-lattices which are directed either in exactly opposite directions (antiferromagnets) or almost opposite each other (weak ferromagnets).

“In the presence of a magnetic field they begin to align with the field, and a total magnetic moment is produced: the higher the magnetic field, the stronger the moment. The elementary magnetic moment of a single atom is sometimes called a ‘spin’. To be more precise, the spin is the angular momentum of the atom’s electron shell, but the related magnetic moment is also called a spin. By the way, besides the electron shell, the atom’s nuclei also has its own magnetic moment, but it is much smaller than the electron shell moment.

“Diamagnetic materials have no intrinsic magnetisation; a magnetic field ‘brings’ the magnetic moment on them by affecting the orbital motion of their atoms’ electrons.

“The magnetization of diamagnets and paramagnets is usually low. That is why ferromagnets are more widely used in technology. Iron is a ferromagnet, the word is derived from the Latin word for iron, ‘ferrum’. Ferromagnets become magnetized in comparatively

low magnetic fields. For example, if you leave a rod made of soft iron oriented from north to south overnight, you will find it magnetized by morning. You can test this with the help of a compass.

“Ferromagnets are magnetized so easily because their atoms initially possess a high magnetic moment. Each piece of a ferromagnet consists of a number of small strongly magnetized areas — small magnets, so to speak. Each area is called a magnetic ‘domain’. Usually, the magnetic moments of these domains are randomly oriented so that the average magnetic moment equals zero.

“As soon as an external magnetic field is applied, the domains begin to align. This alignment produces a net magnetic moment. As the field increases, all the domains line up along the field and the ferromagnet becomes fully magnetized.

“Besides these three simple classes of magnets (i.e. diamagnets, paramagnets, and ferromagnets), there are substances with more complex magnetic properties. These materials have two or more types of magnetic atoms, so there can be two or even more magnetic lattices in a solid crystal. If two sub-lattices have magnetic moments of equal magnitude, but opposite orientation, the material is called anti-ferromagnetic. If the antiferromagnetic sub-lattices are not exactly similar, a net magnetic moment appears. Such a crystal is called ferrimagnetic. In some materials, which resemble antiferromagnets, the crystalline symmetry means that the two sub-lattices are not precisely anti-parallel — they appear to be slightly inclined with respect to each other. This sort of crystal is called a weak ferromagnet.

“There are other more complex magnetic structures. For instance, a certain layer of magnetic moments in the magnetic sub-lattice may not repeat precisely in a neighboring layer, but be slightly turned in comparison to the previous layer.

“Finally, the most complex magnetic material is the ‘spin glass’. In the magnetic spin glass, every elementary magnet interacts with its spin environment in a very complicated way. With respect to certain spins it tries to settle in a parallel way; whereas in respect to

others, it tries to settle in an anti-parallel way. This results in a state with randomly distributed spin directions due to their interaction, not simply because of thermal motion. It turns out that the spin glass is the simplest example of a neural net, i.e. a primitive analog of the neural network of the brain.”

“Why are the magnetic structures so different?” asked Misha. “Does it mean that individual magnetic moments interact just like the magnetic hand of a compass, i.e. their even polarities are pushed away whereas their odd polarities attract each other?”

“No,” answered Sasha. “The atoms inside a crystal possess magnetic moments. They are guided not by classical, but by quantum mechanics. According to quantum mechanics, a turn of each spin results in a deformation of the electron density of a crystal, i.e. it is accompanied by an additional change in the collective energy of all the electrons. The interaction of the spins becomes more complex due to the ‘stirring’ of a crystal’s electron component.

“Thus, the magnetic properties of matter may be very diverse. However, they show nothing similar to mass anisotropy. Therefore, Misha, you either misunderstood something, or you need to ask the Professor how a magnetic field may lead to mass anisotropy.

“Nevertheless, mass anisotropy proper is a very interesting physical phenomenon: I have even realized how to model it. I have a toy at home — a small car with an inertial motor. The motor is a fly-wheel that is driven by means of a gearbox. If you place the car on the floor and start pushing it, the fly-wheel gains momentum and then slowly releases energy sufficient for the car to move. It is also possible first to accelerate and then to restrain the toy, speeding up and slowing down the fly-wheel, respectively. In doing so you develop a feeling that you are handling a rather massive object which is very difficult to accelerate or slow down. Moving this toy in a straight line inevitably drives the fly-wheel up to speed, whereas its lateral motion is as for any other object — so we have a model of an object with mass anisotropy. To get this effect, one can replace smooth wheels by toothed ones and make a corrugated floor. Then, while pushing

the toy across the ribbed floor, its wheels will fit into the grooves and the fly-wheel will be forcefully driven, whereas along the ribs at low friction the toy will move freely with its normal mass. Therefore, we obtain an object with mass anisotropy, the direction of which is preset externally.”

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Playing Billiards

In a week Misha was back in the same hall.

“Today we will play billiards,” said L.A. “Not ordinary billiards, but billiards on a pulsar. All the balls have a strongly anisotropic mass, i.e. their transverse mass $m_{\perp}$ significantly exceeds their longitudinal mass $m_{\parallel}$. For convenience, we will direct the magnetic field horizontally so that the balls will appear light in the longitudinal direction (along the table) and heavy in the transverse one (across the table).

“First we must learn how to hit the balls with a cue. If you hit the ball at an arbitrary angle it will move almost longitudinally. Indeed, if you apply an arbitrary momentum $\mathbf{p}$ to the ball, then resolving it into longitudinal and transverse components gives:

$$\mathbf{p} = \mathbf{p}_{\parallel} + \mathbf{p}_{\perp} = m_{\parallel}\mathbf{v}_{\parallel} + m_{\perp}\mathbf{v}_{\perp}$$

As $m_{\perp} \gg m_{\parallel}$, the transverse speed will be low compared to the longitudinal one, i.e. $v_{\perp} \sim v_{\parallel}m_{\parallel}/m_{\perp}$. To direct the ball in an arbitrary direction, i.e. in order to have $v_{\perp} \sim v_{\parallel}$, one has to hit the ball in such a way that the longitudinal momentum is small, i.e. $p_{\parallel} \sim m_{\parallel}p_{\perp}/m_{\perp} \ll p_{\perp}$. In other words, the cue should be aimed within a small range of angles $\sim m_{\parallel}/m_{\perp}$ from the transverse direction. This is not easy, but it is possible.

“Now let us find out what happens when a moving ball hits a ball at rest. We begin with regular balls on Earth. After a direct hit, the

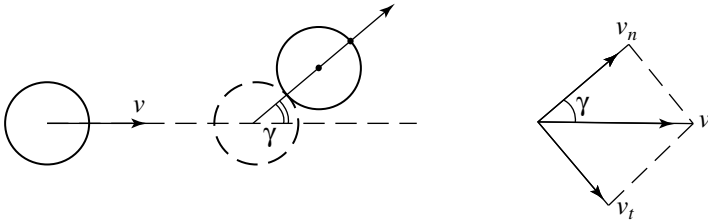


Fig. 5.1 Decomposition of the relative ball velocity $\mathbf{v}$ into tangential, v_t , and normal, v_n , components.

hitting ball stops and the one that was at rest moves with the speed of the hitting ball. This is known from the the conservation laws of energy and momentum — only when the energy and momentum are fully transferred are their net values conserved. Now let's consider an indirect hit (Fig. 5.1). Suppose that at impact the line connecting the centres of the two balls makes angle γ with the velocity of the hitting ball.

“First let us decompose the velocity of the hitting ball into two components: the normal to the tangent plane (at the collision point) $\mathbf{v}_n$ and the tangent component $\mathbf{v}_t$. With respect to the normal component the impact looks like a direct one: the normal momentum of the first ball disappears while the second ball acquires it. The tangent momentum (i.e. the product of the ball's mass and $\mathbf{v}_t$), cannot be transferred to the second ball as there are no tangent forces. So after the collision the balls will move away from each other at a 90° angle: the first ball will move with velocity $\mathbf{v}_t$, and the second will move with velocity $\mathbf{v}_n$.

“Now let us return to billiards on the pulsar. We shall start off with a direct hit: the first ball stops after the collision, and the second one moves with momentum equal to that of the first ball. Geometrically, the direct hit looks as expected only in the case when the balls move precisely across or along the magnetic field. An unusual peculiarity arises in the case of an arbitrary direction of the balls' motion.

“The reason is that, if the momentum of the striking ball is to be completely transferred to the stationary one, the tangent plane

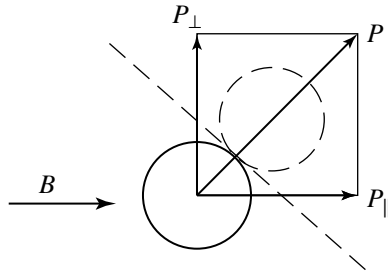


Fig. 5.2 The momentum P of the ball when it experiences a frontal hit with another ball (shown by the dotted line) can be split into a longitudinal and a transverse component.

must be perpendicular to the momentum at the collision point (see Fig. 5.2). Momentum, however, is connected with velocity via the relation:

$$\mathbf{p} = m_{\parallel} \mathbf{v}_{\parallel} + m_{\perp} \mathbf{v}_{\perp}$$

Since $m_{\parallel} \ll m_{\perp}$, $v_{\perp}$ turns out to be much lower than $v_{\parallel}$, as long as $p_{\parallel}$ isn't very small. In other words, most direct hits result in the balls moving at small angles to the magnetic field. It can be said in a different way: if we send the first ball in a direction close to the magnetic field direction, then even after an indirect hit, the second ball will also move at a small angle to the magnetic field. (For the ball to move at a significant angle to the magnetic field, its transverse momentum has to be much higher than the longitudinal one, which is only possible with an almost entirely transverse hit).

“Now let's consider the case of an arbitrary collision. Again the momentum $\mathbf{p}$ can be split into normal $\mathbf{p}_n$ and tangential $\mathbf{p}_t$ components. In an elastic collision, the normal component is transferred to the second ball whereas the tangential one stays with the first ball. Therefore, after the collision, the momenta of the balls are once more perpendicular to each other, and the momentum of the second ball is directed from the point of collision to the centre of the second ball. From the point of view of momenta, the whole picture is the same as in billiards with isotropic balls. However, we observe speeds, not momenta. If we want to control the speed of the second ball, so that

after the hit $v_{\parallel}$ and $v_{\perp}$ for the second ball are of the same order of magnitude, we need it to only gain a small momentum component along the magnetic field. To do so, it should be hit by the first ball in such a way that after the collision it gets a transverse momentum. Thus, the game requires the balls to be hit in a direction almost across the magnetic field (i.e., across the billiards table). This type of play is much more difficult than with isotropic balls so I've had to make the pockets much wider than on a regular table.

“Well, let us start,” said L.A. inviting Misha to the table.

They played several games, until Misha had grown accustomed to the strange behavior of the balls.

“Now I will introduce you to a new game,” said L.A. “which is simpler than billiards, but can only be played with balls of anisotropic mass. The system of ball movement control which we created allows us to simulate not just homogeneous magnetic fields, but also inhomogeneous fields of complex configurations. In particular, we can simulate the magnetic field of a cylindrical conductor with current flowing through it (Fig. 5.3). This magnetic field has an azimuthal direction, so that the magnetic field lines are concentric circles. The field is inversely proportional to the distance from the conductor. The transverse particle mass in a superhigh magnetic field with this

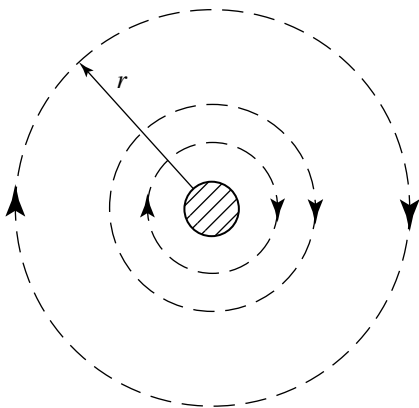


Fig. 5.3 Magnetic field lines of a straight conducting rod with current flowing through it.

geometry looks like $m_{\perp} = m_{\parallel}(1 + a^2/r^2)$, where r is the distance from the conductor, and a characterizes the radius where $m_{\perp} = 2m_{\parallel}$. Roughly speaking, when further away than a from the cylinder, the particle behaves as if it has isotropic mass, and when closer than $r = a$ the anisotropy begins to act; swiftly increasing as the particle moves nearer to the conductor.

“Now let’s throw the ball through this superhigh magnetic field.” L.A. lifted a small ball above the table on which a horizontal conducting rod was affixed, and let the ball go so that it passed close to the conductor. To Misha’s surprise, the ball unexpectedly made a curve as if it were tied to the conductor by an invisible thread. Then it continued its route down along the familiar parabolic trajectory (Fig. 5.4).

“It may seem strange,” said Leonid Andreevich, “but we just saw the ‘free fall’ of a body with anisotropic mass. Certainly, it is not free in the literal sense of the word because the lateral mass of the

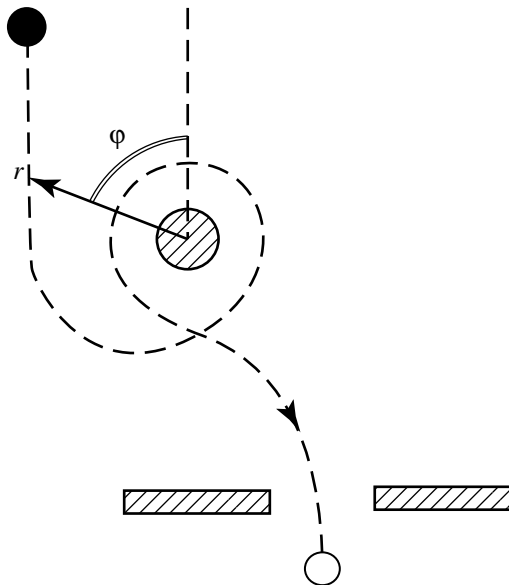


Fig. 5.4 Fall of an anisotropic mass ball through a region containing an azimuthal magnetic field.

body changes along the trajectory. Nevertheless no direct forces are applied to it.

“So, how can all of this be explained? It is convenient to consider the ball’s motion in polar coordinates r, φ .

“Since we released the ball from a height it acquired a high speed. Consequently, in the vicinity of the conductor we can neglect the velocity change caused by gravity, i.e. we can assume the motion to be free. In the field under consideration the mass is conserved during azimuthal motion. Because of the symmetry, i.e. since there are no forces acting in the azimuthal direction, the angular momentum $\mathcal{M} = m_{\parallel} v_{\varphi} r$, where v_{φ} is the azimuthal velocity and is equal to ωr , where ω stands for the angular rotation frequency (the rate at which the angle φ changes), must be conserved. The smaller r is, the larger v_{φ} must be, i.e. the larger the centrifugal force acting in the r -direction.

“It can be said that the body is influenced by the force from a centrifugal potential. The radial movement is based on the fact that the body initially moves to the centre, but under the influence of the centrifugal potential recoils, reflects and flies back. In the case of a particle with isotropic mass, the azimuthal and radial movements combine such that the resultant is a straight trajectory. If, however, $m_{\parallel} \neq m_{\perp}$ and, moreover, $m_{\perp}$ increases as r decreases, the radial movement in the vicinity of the conductor slows down. As a result, the body starts to spiral towards the conductor. Due to the centrifugal potential, the ball stops its movement along the radius and then is pushed out again, leaving the superhigh magnetic field. Depending on the distance from the conductor from which the ball was dropped, i.e. on the ‘impact parameter’, the angle of the ball’s flight from the region of significant mass anisotropy will change.

“The aim of the game is to make the ball penetrate a window located lower than the current conductor (see Fig. 5.4).”

Misha learnt the game easily. After the session he explained to his friends, and, first of all to Sasha, the unusual ways in which anisotropic mass bodies moved.

Sasha was astonished and could not understand what was meant. He had never come across such phenomena in any of his textbooks.

“Do you think I could be introduced to your Professor?” Sasha asked.

“He’s a very kind person,” said Misha, “I’ll ask him next time.”

6

Stubborn Fountains

“Which of two similar-sized bubbles submerged in water do you think will rise faster — an air bubble or a hydrogen one?” asked L.A. at their next meeting.

“The hydrogen bubble, of course” said Misha without thinking, “hydrogen is lighter than air.”

“This time you have been too fast,” said L.A. with a smile. “Let us think more thoroughly.

“According to Archimedes’ principle, the upwards force on the bubble is $F_A = \rho g V$, where ρ is the density of the liquid and V the volume of the bubble. In this case, according to Newton’s law we have:

$$ma = F_A - F_D$$

Here a is the bubble’s acceleration, and F_D is the drag force experienced by the bubble while moving in the liquid. The drag increases with the speed, so the motion of the bubble is described as follows: at the moment of the bubble’s take-off from the point of its formation (e.g. if it is found on the bottom of a tea-pot or if it is slowly expelled from a thin tube in an aquarium) its speed is close to zero, so the drag force is small and the bubble accelerates upwards. As the speed grows, F_D grows too until we have equilibrium between F_D and F_A , so the speed of the bubble becomes constant. F_D and, therefore, the speed of the steadily rising bubble depend on the liquid’s

viscosity: the same bubble in honey, for example, rises much more slowly than in water. Both F_D and the Archimedes force F_A , depend on the bubble's size such that small bubbles rise more slowly than large ones.

“Let's assume, for the sake of simplicity, that the viscosity is vanishingly small. Then which mass is m ? It would seem that $m = \rho'V$ where ρ' is the density of the gas in the bubble, but this is not true: this assumption contradicts our routine observations. The density of air is a thousand times lower than that of water and, therefore, the air bubble in the water would be accelerated by a thousand times g , which is certainly incorrect. To understand what is meant by the mass m , let us consider how the bubble is rising. If the bubble moves with speed v , the liquid streams around the bubble with a velocity proportional to v (see Fig. 6.1). Therefore the Archimedes force, whilst lifting the bubble, also makes the liquid around the bubble move.

“For further deliberations it is convenient to use energy since energy is being conserved (if we also consider the kinetic energy transformed into heat due to the viscosity).

“It is obvious that the liquid which streams around the bubble has a given kinetic energy E which increases as the speed of the

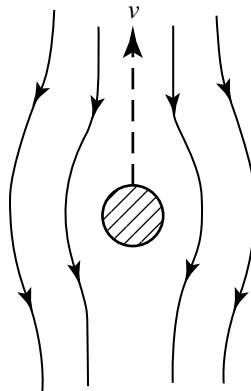


Fig. 6.1 The water stream around the gas bubble.